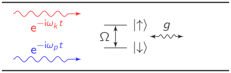

Qubit-Photon Interactions in Waveguides

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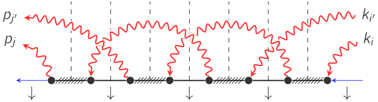
Hacettepe Üniversitesi

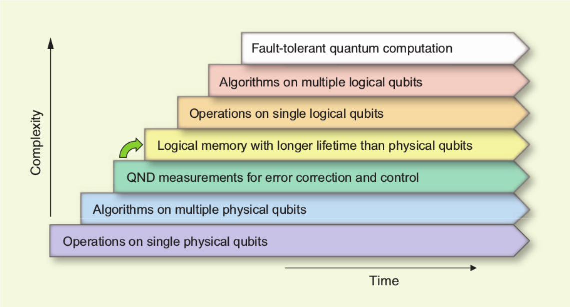


Outline

Why Quantum Information Processing?

- Algorithms exist that enable very fast computation using entangled states
  - Factoring large numbers
  - Searching big data
  - Solving a large number of equations
- Secure communications
- Simulate quantum many-body systems

Stages in Quantum Computing

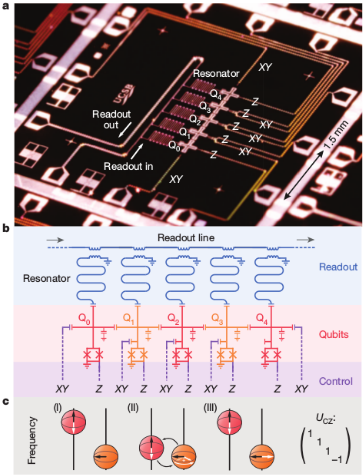


- Lowest two levels: physical layer
- Physical limits on coherence time $\rightarrow$ error correction[1]
- Use of quantum error correction to achieve quantum memory with longer coherence than physical subsytems (not yet done)

[1] Devoret and Schoelkopf, Science **339**, 1169–1174 (2013).

Circuit QED Implementations

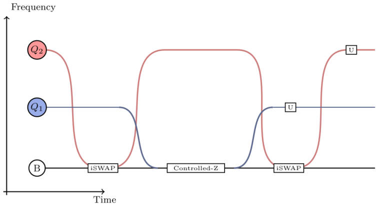
- Josephson junction based qubits[2]
- Connections by microwave transmission lines
- Cooled down to millikelvin temperatures
- Qubit energy levels and their coupling to resonators are controlled with external signals



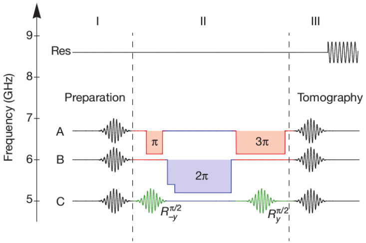
[2]Barends, Nature **508**, 500–503 (2014).

Formation of Gates

- Gates can be formed by tuning energy levels of qubits[3]



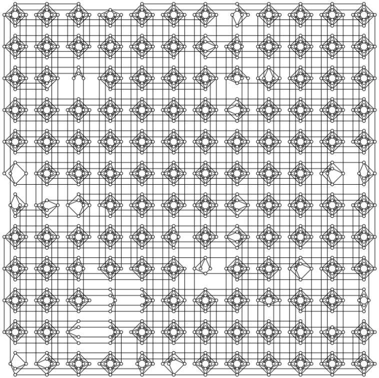
- Or by external microwave pulses which excite certain transitions among different energy levels[4]



[3] Egger and Wilhelm, Superconductor Science and Technology **27**, 014001 (2014).
[4] Fedorov, Nature **481**, 170–172 (2012).

Commercial Quantum Annealing Processor: D-Wave

- Specialized system, implements quantum annealing only[5]
- Degree of 'quantumness' being discussed
- Dec 8, 2015 "[Quantum annealing] is more than 10^8 times faster than simulated annealing running on a single core."[6]

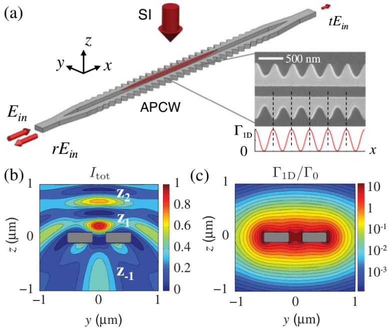


- Example hardware graph of 12×12 units, each with 8 qubits, some are defective

[5] King, (2015), arXiv:1508.05087.
[6] Denchev, (2015), arXiv:1512.02206.

Quantum Optics Based on Nanophotonics

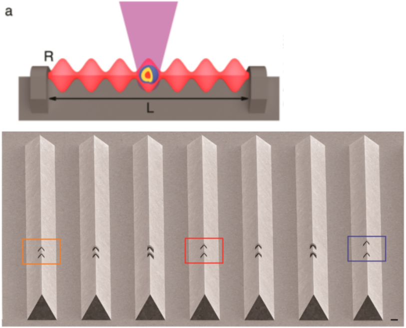
- Photonic crystal waveguide with two supported modes[7]
- One mode used to trap ions
- Other mode used to communicate with ion based qubits



[7] Goban, Phys. Rev. Lett. **115**, 063601 (2015).

Quantum Optics Based on Plasmonics

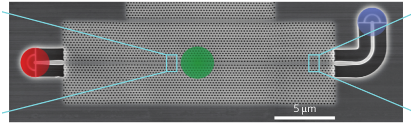
- Plasmonic wedge waveguide cavity[8]
- Quantum dots are printed on the wedge
- External laser light used to pump quantum dot, emission into waveguide mode



[8] Kress, Nano Letters **15**, 6267–6275 (2015).

Quantum Optics Based on Photonic Crystals

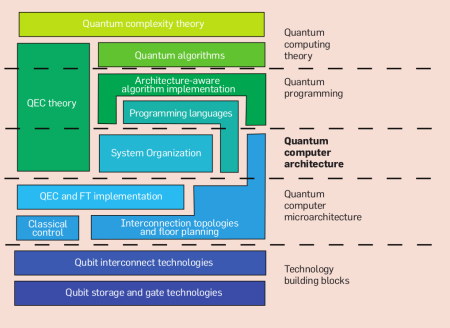
- Defect line in a photonic crystal forms a waveguide[9]
- Quantum dot grown on In(Ga)As layer
- Input and output gratings used to couple light in and out



[9] Söllner, Nat Nano **10**, 775–778 (2015).

Focus on Modeling of Physical Layer

- Gates do operations on 'stationary' qubits[10]
- Results should be transferred by 'flying' qubits
- Interconnects are an integral part of the design
- Remainder of the talk on recent advances in understanding photon - qubit interactions in waveguides

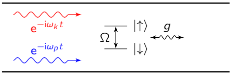


QEC: quantum error correction
FT: fault tolerant

[10] Van Meter and Horsman, Commun. ACM **56**, 84–93 (2013).

System Details

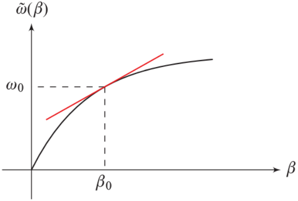
- Qubit positioned in an anti-node of the waveguide mode



- Multi-mode Jaynes-Cummings Hamiltonian under RWA and dipole approximation

$$H = \underbrace{\int \mathrm{d}k \omega_k a_k^\dagger a_k + \frac{\Omega}{2}\sigma_z}_{H_0} + \underbrace{g \int \mathrm{d}k (\sigma^+ a_k + a_k^\dagger \sigma^-)}_{V}$$

- Linearized dispersion relation *(for now)*



Qubit as a Perfect Single Photon Mirror

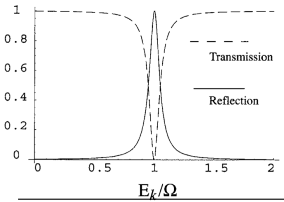
- Hamiltonian re-written in real space
- Stationary state written as

$$|E_k\rangle = \int \mathrm{d}x \left[\phi^\dagger_{k,R}(x) c^\dagger_R(x) + \phi^\dagger_{k,L}(x) c^\dagger_L(x)\right] |0\rangle + e_k \sigma^+ |0\rangle$$

$$\phi^\dagger_{k,R}(x) = \exp(\mathrm{i}kx)\theta(-x) + t\exp(\mathrm{i}kx)\theta(x)$$

$$\phi^\dagger_{k,L}(x) = r\exp(-\mathrm{i}kx)\theta(-x)$$

- Solve for $H|E_k\rangle = E_k|E_k\rangle$
- Transmission and reflection of single photon[11]



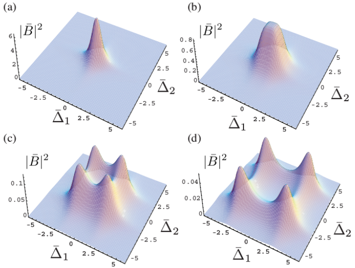
More than one qubit doable via transfer matrix approach

[11] Shen and Fan, Opt. Lett. **30**, 2001–2003 (2005).

Non-Linearity by Two-Photons

- Qubit cannot absorb two photons → non-linearity
- Complicated stationary states (Bethe ansatz)[12]
- Formation of non-separable two-photon states

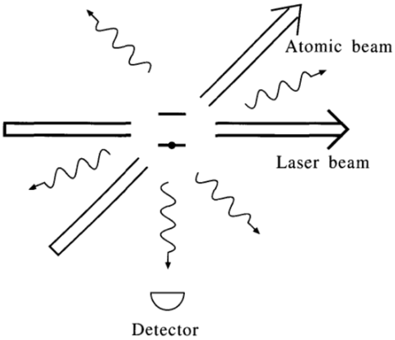
$$\langle k_{2,R} p_{2,R} | S | k_{1,R} p_{1,R} \rangle = t_{k_1} t_{p_1} [\delta_{k_1,k_2} \delta_{p_1,p_2} + \delta_{k_1,p_2} \delta_{p_1,k_2}] + \frac{1}{4} B \delta_{k_1+p_1, k_2+p_2}$$



$k_1 + p_1 - 2\Omega$ starts as 0 in (a) and increases in (b)–(d)

[12] Shen and Fan, Physical Review A **76**, 062709 (2007).

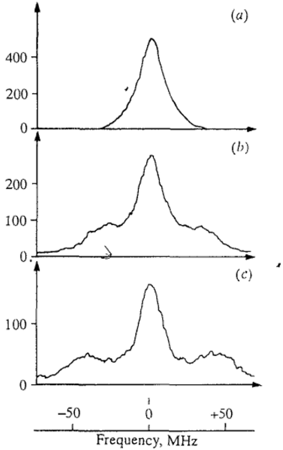
What is Resonance Fluorescence?



from Scully & Zubairy, *Quantum optics*, Cambridge University Press, 1997, p. 292

- Excite atoms with a laser light *(coherent state)*
- Observe in an orthogonal direction
- Control signals are in coherent states, important to analyze qubit-coherent state interaction in waveguides

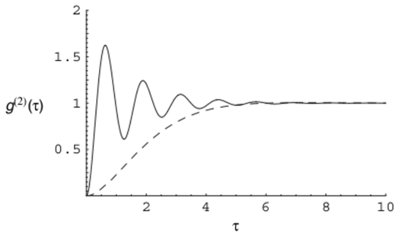
Mollow Triplets



from Mandel, L. & Wolf, E. *Optical coherence and quantum optics*, Cambridge University Press, 1995, p. 785

- Light emitted from the atom has coherent (i.e. same freq) and incoherent components (i.e. different freq)
- High excitation intensity → Rabi oscillations → Mollow triplet

$g^{(2)}$ Function in Resonance Fluorescence

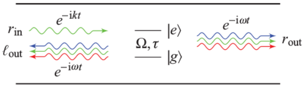


from Walls, D. & Milburn, G. J. *Quantum Optics*, Springer, 2008, p. 208

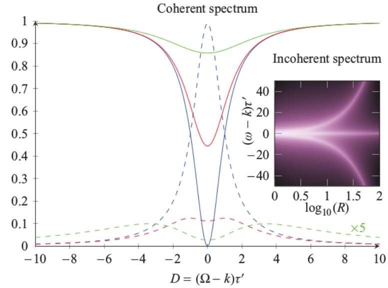
- $g^{(2)}(\tau)$ gives the probability of observing two photons with delay τ at the detector
- Rabi oscillations at higher intensity $\rightarrow$ oscillations in $g^{(2)}$
- Anti-bunching as seen from $g^{(2)}(\tau = 0) = 0$

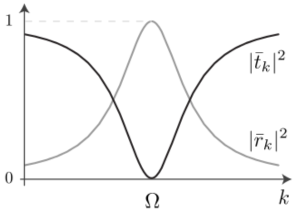
Resonance Fluorescence in Waveguides — Spectrum

- Scattering of a coherent state with varying intensity from a qubit[13]



- Input-output formalism[14] used, much faster analysis

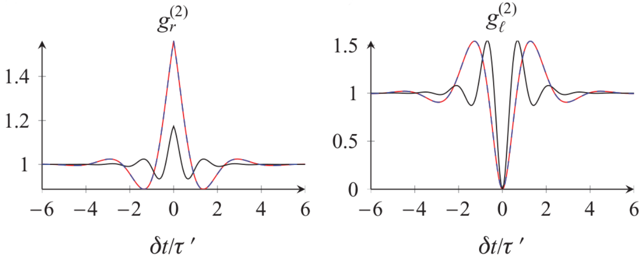




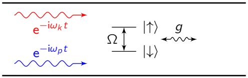
[13] Kocabaș, Rephaeli, and Fan, Phys. Rev. A **85**, 023817 (2012).
[14] Fan, Kocabaș, and Shen, Phys. Rev. A **82**, 063821 (2010).

Resonance Fluorescence in Waveguides — $g^{(2)}$

- Bunching for transmitted light
- Anti-bunching for reflected light
- Interference of radiated fields in forward/backward directions



Dispersive Modes of Waveguides — Tight Binding Case



- Multi-mode Jaynes-Cummings Model
- Tight-binding lattice dispersion $\omega_k = -2J\cos k$, extension to other types of dispersion also possible

$$H = \underbrace{\int_{-\pi}^{\pi} \mathrm{d}k\,\omega_k a_k^\dagger a_k + \frac{\Omega}{2}\sigma_z}_{H_0} + \underbrace{g\int_{-\pi}^{\pi} \mathrm{d}k(\sigma^+ a_k + a_k^\dagger \sigma^-)}_{V}$$

- Analyses will be made via the use of the resolvent

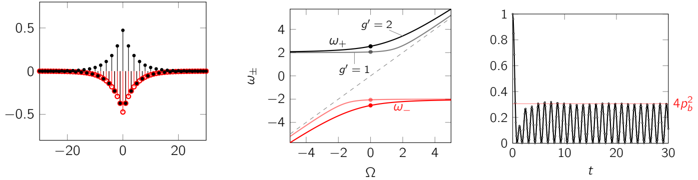
$$G(z) = \frac{1}{z - H}$$

Bound Modes

- We need the following matrix elements of the resolvent

$$\langle\uparrow|G(z)|\uparrow\rangle \equiv G_1(z) \qquad \langle k\downarrow|G(z)|\uparrow\rangle \equiv G_2(z;k)$$
$$\langle\uparrow|G(z)|k\downarrow\rangle \equiv G_3(z;k) \quad \langle p\downarrow|G(z)|k\downarrow\rangle \equiv G_4(z;p,k)$$

- The poles of $G_1(z)$ define *bound modes*, i.e. polaritonic modes, of the system[15]



- There are two bound modes with energies $\omega_\pm$ outside the free-photon energy range $(-2J, 2J)$

$$|\Psi_\pm\rangle = \sqrt{p_b}|\uparrow\rangle + \sqrt{p_b}g\int_{-\pi}^{\pi} \mathrm{d}k \frac{|k\downarrow\rangle}{\omega_\pm + 2J\cos k}$$

[15] Lombardo, Ciccarello, and Palma, Phys. Rev. A **89**, 053826 (2014).

One-Photon Scattering

- Scattering matrix elements can be calculated via $T(z)$

$$\langle p{\downarrow}|S|k{\downarrow}\rangle = \langle p|k\rangle - 2\pi \mathrm{i}\,\delta(\omega_p - \omega_k) \lim_{\eta\to 0^+} \langle p{\downarrow}|\,T(\omega_p - \Omega/2 + \mathrm{i}\eta)|k{\downarrow}\rangle$$

- It is easy to get $T(z)$ from $G(z)$ via the following relationships

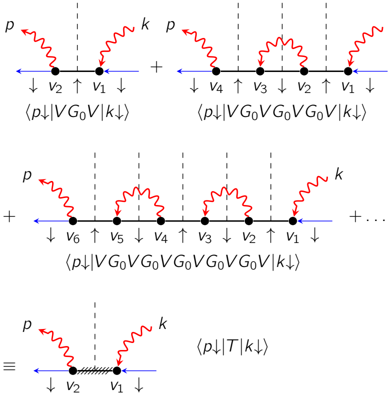
$$T(z) = V + VG(z)V \quad \text{and} \quad G(z) = G_0(z) + G_0(z)\,T(z)G_0(z)$$

Feynman Diagram Representation

- From the Lippmann-Schwinger equation $T(z) = V + VG_0(z)T(z)$ we get

$$T(z){=}V{+}VG_0(z)V{+}VG_0(z)VG_0(z)V{+}VG_0(z)VG_0(z)VG_0(z)V{+}\dots$$

- The series representation leads to the Feynman graphs[16]



[16] Kocabaş, Phys. Rev. A **93**, 033829 (2016).

Two-Photon Scattering in a Dispersive Waveguide

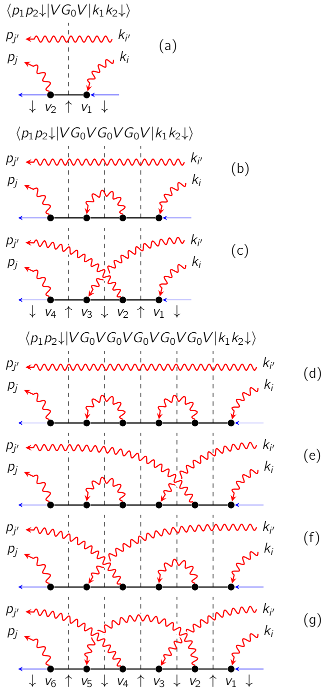
- Two-photon calculations require the following matrix elements

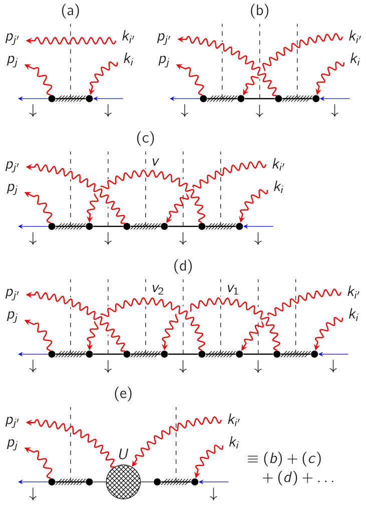
$$\langle p\uparrow|G(z)|k\uparrow\rangle \equiv G_5(z;p,k) \qquad \langle p_1 p_2\downarrow|G(z)|k\uparrow\rangle \equiv G_6(z;p_1,p_2,k)$$
$$\langle p\uparrow|G(z)|k_1 k_2\downarrow\rangle \equiv G_7(z;p,k_1,k_2) \qquad \langle p_1 p_2\downarrow|G(z)|k_1 k_2\downarrow\rangle \equiv G_8(z;p_1,p_2,k_1,k_2)$$

- G_6, G_7, G_8 can be written in terms of G_5
- Analysis based on solvable Lee model of QFT
- We derive an integral equation for G_5

$$U(z;p,k) = \frac{1}{z-\omega_p-\omega_k} + g^2 \int_{-\pi}^{\pi} \mathrm{d}p_i \frac{U(z;p_i,k)}{H(z;p_i)(z-\omega_p-\omega_{p_i})}$$

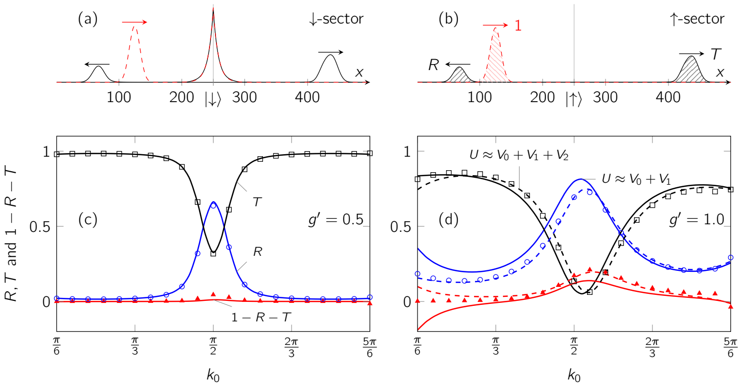
- U function can be described in terms of Feynman graphs





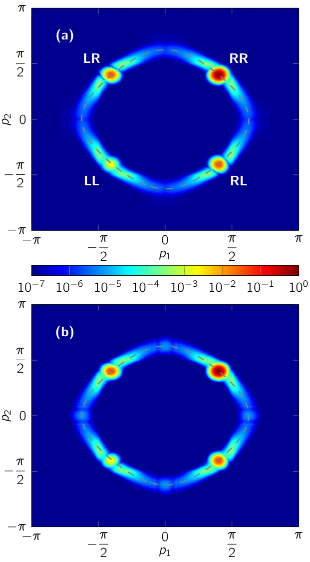
Two photons lead to many possible diagrams

Test of Feynman Diagram Calculations — Bound Modes



- Free photon coming and scattering off of a bound mode
- Much better agreement as number of diagrams increased for large g

Test of Feynman Diagram Calculations — Free Modes

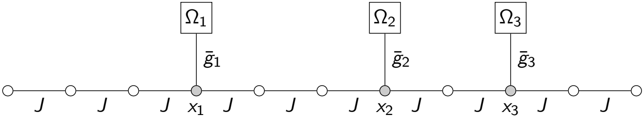


- Two-photon scattering calculated with Feynman diagrams (up) and independent numerical calculations (down)
- Initial state is two right going photons, final state is plotted on the left
- R: right, L: left, RR: two right going photons

Numerical Time Evolution Studies

Developed a numerical algorithm to evaluate the time evolution of multi-qubit systems with the Hamiltonian

$$H = -J\sum_{i=1}^{L-1}\left(a_{i+1}^{\dagger}a_i + a_i^{\dagger}a_{i+1}\right) + \sum_{s=1}^{n}\frac{\Omega_s}{2}\sigma_{z_s} + \sum_{s=1}^{n}\bar{g}_s\left(\sigma_s^{+}a_{x_s} + a_{x_s}^{\dagger}\sigma_s^{-}\right)$$

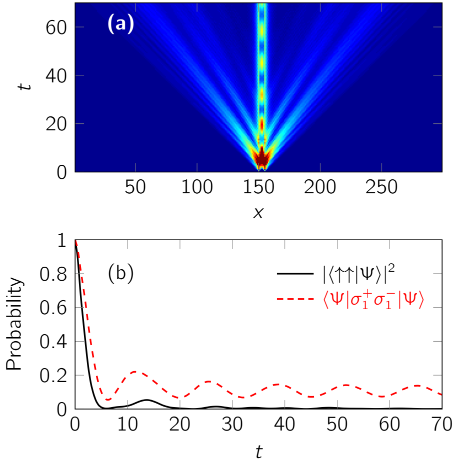


for an arbitrary initial state.[17]

[17] Kocabaş, (2016), arXiv:1603.02920.

Time Evolution of $|\uparrow\uparrow\rangle$ State

- Analytic calculations quickly get quite complicated
- Multi-qubit, multi-photon calculations are of interest for gate design
- Numerical time evolution via Krylov-subspace method
- $|\Psi(t)\rangle = \mathrm{e}^{-\mathrm{i}Ht}|\uparrow\uparrow\rangle$
- Excitation of a two-qubit bound mode, with a bouncing photon between qubits



Bound States In the Continuum (BIC)

- Similar to the single qubit case, multi qubit systems support bound modes, too.[18]
- Unlike the single qubit case, some multi qubit bound states are within the propagating photon energy continuum range $\rightarrow$ BIC
- For two qubits we have[19]

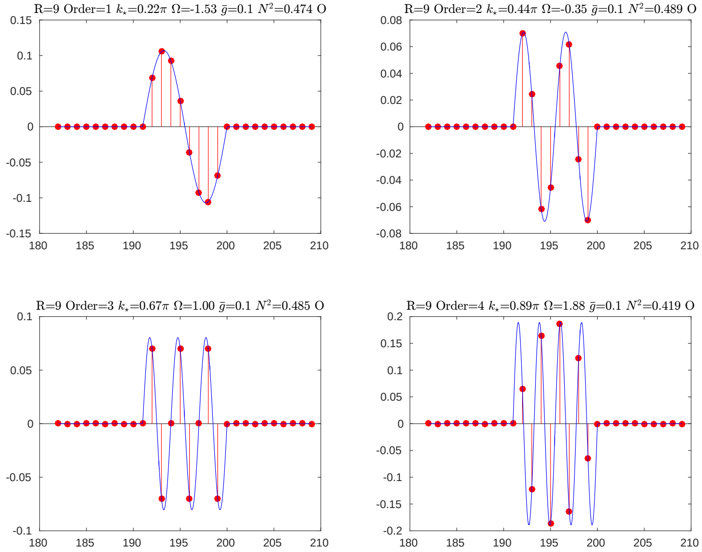
$$|\Psi^{\pm}_{n_{e,o}}\rangle = \mathcal{N}\left[|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle + \sum_x (-i)\bar{g}\frac{e^{ik_\star|x+\frac{R}{2}|} \pm e^{ik_\star|x-\frac{R}{2}|}}{\sqrt{4J^2-\Omega^2}}|x\downarrow\downarrow\rangle\right]$$

$$\mathcal{N} = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+\frac{\bar{g}^2 R}{4J^2-\Omega^2}}},$$

[18] Calajo, (2015), arXiv:1512.04946.
[19] Kocabaș, (2016), arXiv:1603.02920.

BIC Shapes

- Condition for BIC is $1 \pm e^{ik_\star R} = 0$ and $\Omega = -2J\cos k_\star$
- For $\bar{g} \ll 1$ we have $\mathcal{N}^2 \approx 0.5$



DFS Based Logical Qubits

- One can associate the BIC with decoherence-free states[20] from the Dicke model
- There are quantum gate designs that make use of decoherence-free states (DFS)[21]
- Two qubits make one *logical* qubit with

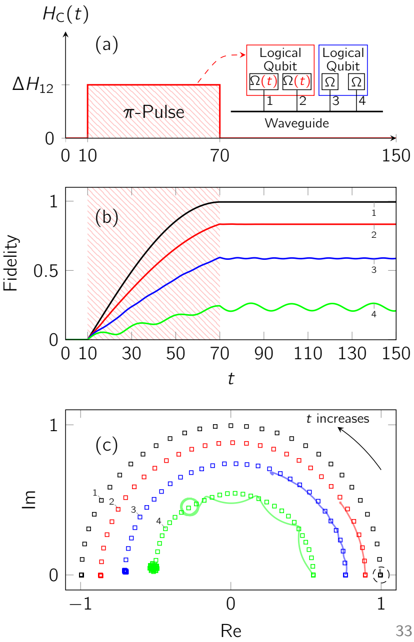
$$|0\rangle \equiv |\downarrow\downarrow\rangle$$
$$|1\rangle \equiv |\Psi^{-}_{n_\circ}\rangle \approx \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

[20] Chen, Yang, and An, (2016), arXiv:1601.02303.

[21] Paulisch, Kimble, and Gonzalez-Tudela, (2015), arXiv:1512.04803.

Pauli (-Z) Gate

- We use four physical qubits to get two logical qubits
- Initial state is $(|10\rangle + |01\rangle)/\sqrt{2}$
- We apply a π-pulse to arrive at the final state $(|10\rangle - |01\rangle)/\sqrt{2}$
- $\Delta H_{12} = \frac{\Delta}{2}(\sigma_{z_1} + \sigma_{z_2})$
- (set 1) $R = 4, \bar{g} = 0.1, n_o = 1$
  (set 2) $R = 4, \bar{g} = 0.5, n_o = 1$
  (set 3) $R = 7, \bar{g} = 0.5, n_o = 1$
  (set 4) $R = 7, \bar{g} = 0.5, n_o = 3$



Summary

- Waveguide integrated qubits are now technologically possible
- Rich collection of effects arising from photon-qubit interactions in waveguides
  - Particularly when waveguide dispersion is taken into account

- Outlook
  - Waveguide + qubits system is a promising simulation platform for many-body physics[22]
  - It seems also feasible to investigate topological order with synthetic dimensions excited in waveguide + qubits[23]

[22] Douglas, Nat Photon **9**, 326–331 (2015).
[23] Graß, Phys. Rev. A **91**, 063612 (2015).

Thank you for your attention.

Happy to share the slides if you are interested, email me at ekocabas@ku.edu.tr to get a copy.