Non-equilibrium Fractional Hall Response After a Topological Quench

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F. Nur Ünal, Erich Mueller, MÖO, PRA **94**, 053604 (2016), arXiv:1608.04395

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Outline

genus=0

Quench

genus=1

"Universal"
Fractional
Hall response

Outline

genus=0

Quench

genus=1

"Universal" Fractional Hall response

- Motivation
- Non-equilibrium topological response
    - Chern number ←?→ Hall conductivity
- Fractional Non-equilibrium Hall response
    - Single Dirac cone
    - Haldane model
- Experimental considerations; (non)linear response, finite size, trap…

- Topological Order = Interesting!
  - Equilibrium…

- Topological Order = Interesting!
  - Equilibrium well studied

✓ Local perturbations ? Global perturbation

- Penn State – Rigol, Nature Comm. 2015
- Cambridge – Cooper, PRL 2015
- Max Plank, NYU – Oka, Mitra, PRB 2015
- Göttingen – Kehrain, PRB, 2016
- Innsbruck – Zoller, PRL 2016
- Cornell, Bilkent –Mueller, Oktel (PRA) 2016
- CalTech – Refael, preprint 2016
- Hamburg – Sengstock, Weitenberg, preprint (2016)
- Tsinghua – Zhai, preprint (2016)
- ETH, UIUC, ICFO, Rutgers, Rice, Colorado…

❖ Cold atoms play an important role in these developments!

- Hamburg, non-equilibrium distribution ✓.
- Zurich, in progress …

- Topological Order = Interesting!
  - Equilibrium…

✓ Local perturbations ? Global perturbation

? Dynamical response: All topological transitions non-adiabatic.

? Time-dependent transition between topological states: Do invariants survive?

✓ Experimentally relevant

- What happens when a system is suddenly driven between two topologically different phases?
  - Chern number is a property of the wavefunction! (Equilibrium: band=wavefunction)

$$C = \frac{1}{2\pi} \int dk_\mu \, dk_\nu \, \Omega_{\mu\nu}$$

(μ, ν) – parametrizes momentum space 2-manifold

- What happens when a system is suddenly driven between two topologically different phases?
  - Chern number is a property of the wavefuncion! It is preserved!

$$C = \frac{1}{2\pi}\int dk_\mu \, dk_\nu \, \Omega_{\mu\nu}$$

(μ, ν) – parametrizes momentum space 2-manifold

$$\Omega_{\mu\nu} = \partial_{k_\mu} A_{k_\nu} - \partial_{k_\nu} A_{k_\mu}$$

$$A_{k_\mu} = i \left\langle \Psi \middle| \partial_{k_\mu} \middle| \Psi \right\rangle$$

- Two-band model, e.g. Dirac Hamiltonian
  - Initially filled lower band, excite some particles to upper band

$$\Psi(k,\theta,t) = a(k,t)\,|\ell(k,\theta)\rangle + b(k,t)\,|u(k,\theta)\rangle$$

eigenstates of the final Hamiltonian

Cooper, PRL'15

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  - Chern number = integer
  - Just after the quench, Ψ does not have time to respond
  - In later times, unitary evolution does not change C

Cooper, PRL'15

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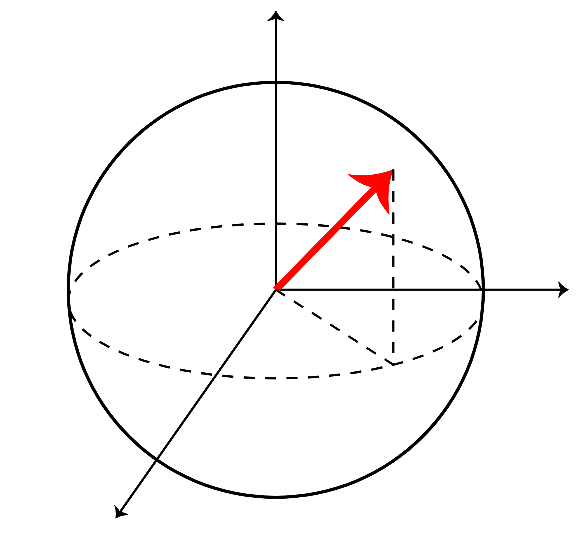

# of wrappings = C

- E.g. Pseudo-spins on Bloch sphere for a Dirac cone

$$\mathcal{H}\left(\vec{k}\right) = \begin{pmatrix} \Delta & cke^{i\theta} \\ cke^{-i\theta} & -\Delta \end{pmatrix} \qquad H_k = -\frac{1}{2}\vec{h}_k \cdot \vec{\sigma}$$

Quench = Effective magnetic field changes at each k

Rigol, Nature’15

- What happens when a system is suddenly driven between two topologically different phases?
  - Chern number is preserved!

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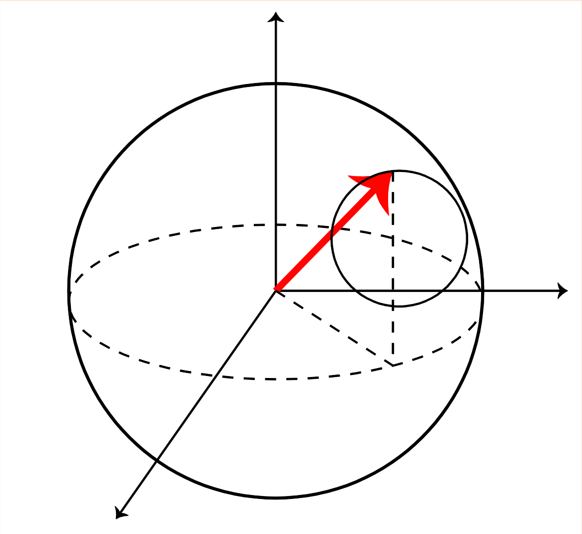

* spins precess around $\vec{h}_k$

* preserves topological character of the initial configuration

* # of wrappings = same

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Rigol, Nature'15

$$C = \frac{1}{2\pi}\int dk_{\mu}\, dk_{\nu}\, \Omega_{\mu\nu}$$

Quench, then unitary evolution…

- *Pure state → Pure state*
- *Chern number is the property of the wave function and preserved!*
- *BUT SO WHAT?*

Take home message # 1

- How to classify the resulting non-equilibrium state then?
- C=0, quench, still C=0, but edge currents??
- What would a Hall Bar experiment yield after the quench?
- Observables: Hall conductivity

$$\sigma_H^{neq} \neq C\,\frac{e^2}{h} = \cdots$$

σ_H is integer, assuming a ground state !

to see what it is out-of-equilibrium

- Calculate transverse current

Hall conductivity: $\sigma_H^{neq} = ?$

Semiclassically

Equation of motion: $\frac{d\vec{p}}{dt} = e\vec{E} + \frac{e}{c}\frac{d\vec{r}}{dt} \times \left(\vec{\nabla}_r \times \overrightarrow{A}(\vec{r})\right)$

$$\frac{d\vec{r}}{dt} = \vec{\nabla}\varepsilon(\vec{p}) + \hbar\frac{d\vec{p}}{dt} \times \left(\vec{\nabla}_p \times \overrightarrow{\mathcal{A}}(\vec{r})\right)$$

$j = env$

$$\sum_{\vec{p}} v_p \approx \int d\vec{p} \left\langle \Psi \middle| \left\{ \vec{\nabla}\varepsilon(\vec{p}) + \hbar \underbrace{\frac{d\vec{p}}{dt}} \times \left(\vec{\nabla}_p \times \overrightarrow{\mathcal{A}}(\vec{r})\right) \right\} \middle| \Psi \right\rangle$$

Roy et. al, PRA'15

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$$= \int d\vec{p} \left\langle \Psi \middle| \left\{ \vec{\nabla}\varepsilon(\vec{p}) + \hbar \left(e\vec{E} + \frac{e}{c}\vec{v}\,\mathrm{x}\vec{B} \right) \mathrm{x} \underbrace{\left(\vec{\nabla}_p \mathrm{x} \vec{\mathcal{A}}(\vec{r}) \right)}_{\vec{\Omega}_{eq}} \right\} \middle| \Psi \right\rangle$$

Transverse drift $\int (a^* \langle \ell | + b^* \langle u |) \left(\vec{E} \; x \; \overrightarrow{\Omega}_{eq} \right) (a \, |\ell\rangle + b \, |u\rangle)$

$\sim$

$$\sigma_H^{neq} = \frac{e^2}{2\pi h} \sum_{n} \int d^2k \; P_n(\vec{k}) \;\; \Omega_n(\vec{k})$$

Roy et. al, PRA'15

Quantum Mechanically

- Or calculate the transverse current…

$$\sigma_H^{neq} = \frac{e^2}{2\pi h} \sum_{\substack{n \\ bands}} \int d^2k \, P_n(\vec{k}) \;\, \Omega_n(\vec{k}) = \frac{e^2}{h} C_{neq}$$

*Diagonal ensemble!

$\vec{E}(t) = E_0 \cos\omega t \;\, \hat{x}$, $\hbar\dot{k} = -eE(t)$, take $\omega \to 0$ for DC Hall conductivity…

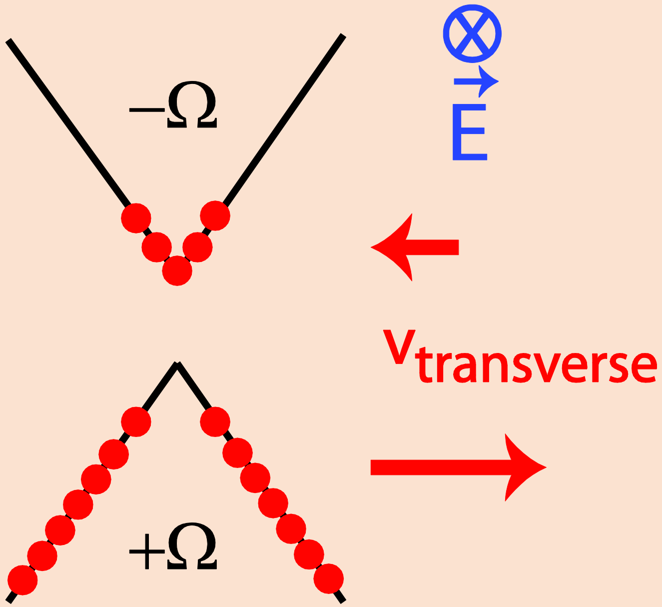
$$\mathcal{H}(t) = \sum_k H_k \cong \sum_k H_k\Big|_{k=0} + \hat{v}_k.\left(-e\vec{E}/\omega \, sin\omega t\right)$$

$|\Psi_k(t)\rangle = \sum_{b=\ell,u} c_{bk}(t) e^{-i\omega_{bk}t} \, |b,k\rangle$, $c_{bk}(t)$=… time-dependent perturbation theory

$\langle J_y \rangle = \sum_{\vec{k}} \langle \Psi_k(t) | -e\hat{v}_y | \Psi_k(t) \rangle$ Fast oscillating terms die out…

- Quench between topologically different phases:

$$\sigma_H^{neq} = \frac{e^2}{2\pi h} \sum_{\substack{n \\ bands}} \int d^2k \, P_n(\vec{k}) \;\; \Omega_n(\vec{k}) = \frac{e^2}{h} \, C_{neq}$$



- Not quantized!
- Not universal!

$P_n(\vec{k})$: Probability distributions

$\Omega_n(\vec{k})$: Berry Curv. of the bands

C_{neq}: Dimensionless Hall conductivity

Usual TKNN for the ground state

Take home message # 2

$$\sigma_H^{neq} = \frac{e^2}{2\pi h} \sum_{\substack{n \\ bands}} \int d^2k \; P_n(\vec{k}) \;\; \Omega_n(\vec{k}) \quad \neq C$$

can be anything!

Take home message # 2

$$\sigma_H^{neq} = \frac{e^2}{2\pi h} \sum_{\substack{n \\ bands}} \int d^2k \; P_n(\vec{k}) \;\; \Omega_n(\vec{k}) \quad \neq C$$

can be anything!

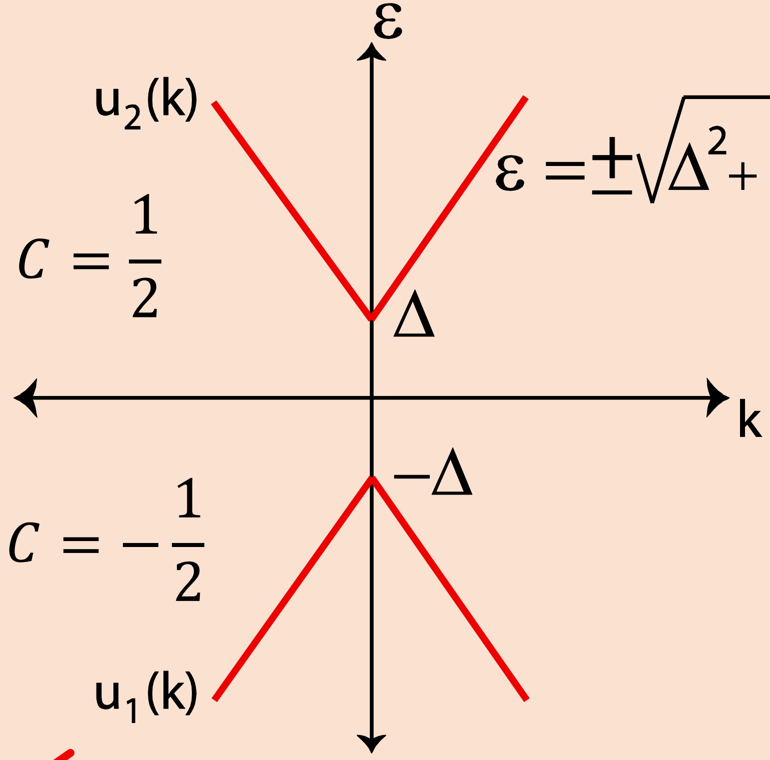
Let's consider a specific system...

- Near transition; low-energy physics ~ massive Dirac model

Static solutions

$$\mathcal{H}\left(\vec{k}\right) = \begin{pmatrix} \Delta & cke^{i\theta} \\ cke^{-i\theta} & -\Delta \end{pmatrix} \qquad ke^{i\theta} = k_x + ik_y$$

$$\mathcal{H}_k\, u(k) = \varepsilon_k\, u(k)$$



$$\varepsilon = \pm\sqrt{\Delta^2 + c^2k^2}$$

$$\Omega = \frac{\pm\, c^2\Delta}{2(\Delta^2 + c^2k^2)^{3/2}}$$

depends on the sign of Δ

independent of the sign of Δ

- Near transition; low-energy physics ~ massive Dirac model

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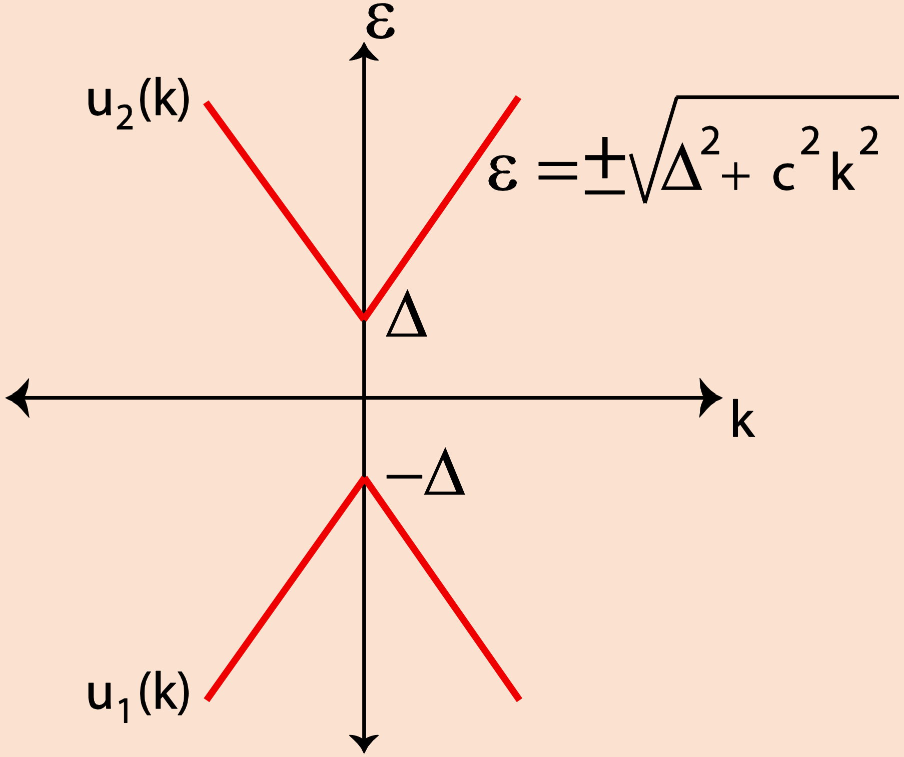

- Non-interacting fermions,

initially ground state at half filling

$$C_{initial} = -\frac{1}{2}$$

- Invert the sign of the mass $\Delta \rightarrow -\Delta$

Occupation Probabilities: $P_1(k) = \frac{c^2k^2}{\Delta^2 + c^2k^2}$ $P_2(k) = \frac{\Delta^2}{\Delta^2 + c^2k^2}$

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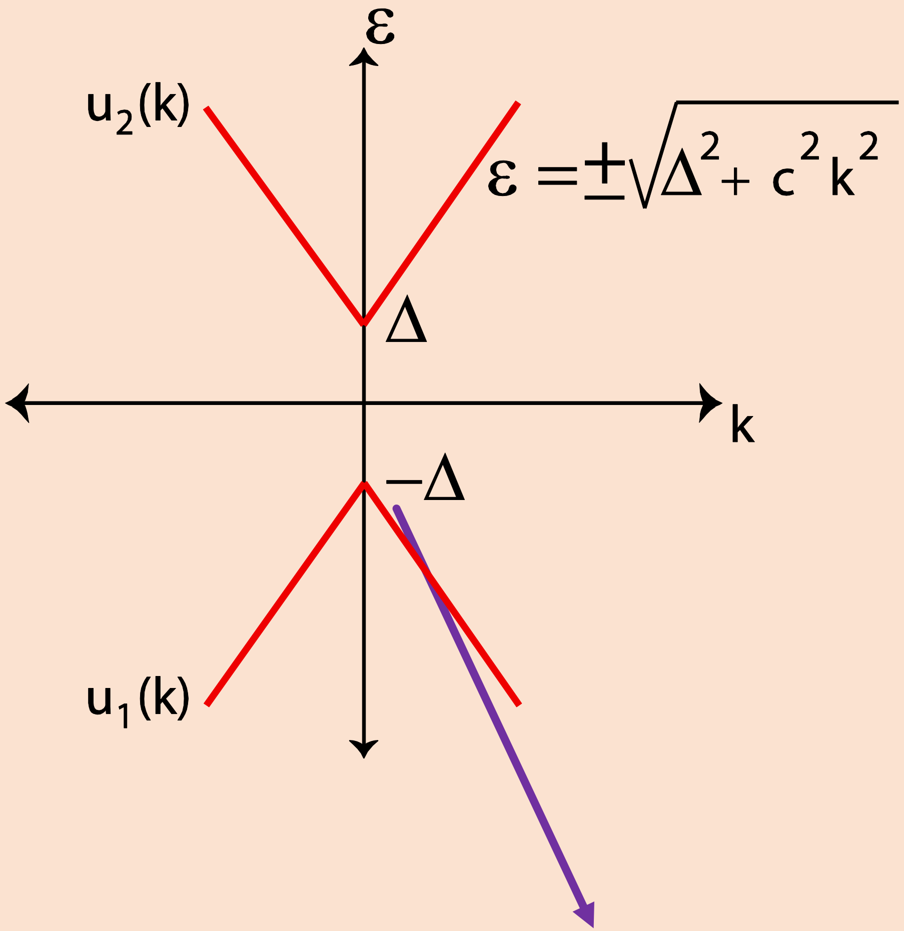

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The summit matters the most!

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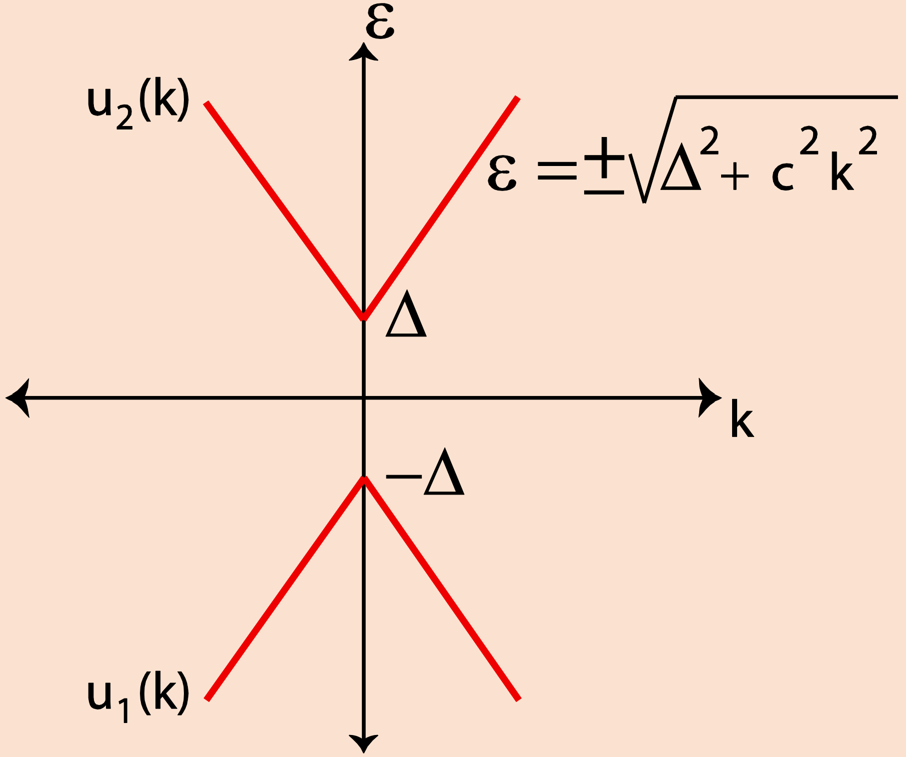


$$C_{neq} = \frac{1}{2\pi} \sum_{\substack{n \\ bands}} \int d^2k\, P_n(\vec{k})\ \Omega_n(\vec{k})$$

$$C_{neq} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$$

Lower band — Upper band

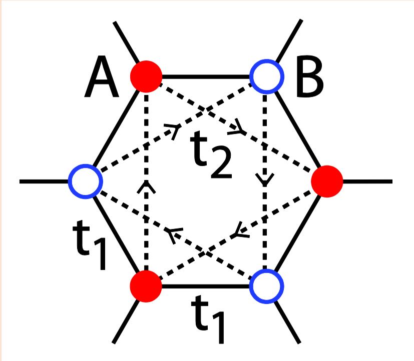
❖ For symmetric quenches, initially $-\frac{1}{2}$, final $\frac{1}{6}$ => change always $\frac{2}{3}$

Take home message # 3

- For a single Dirac cone,
  in a mass-sign-inverting quench,
  the symmetry between the initial and final energy spectrum yields
  a universal (independent of Δ,c) fractional Hall conductivity $\frac{1}{6}$.

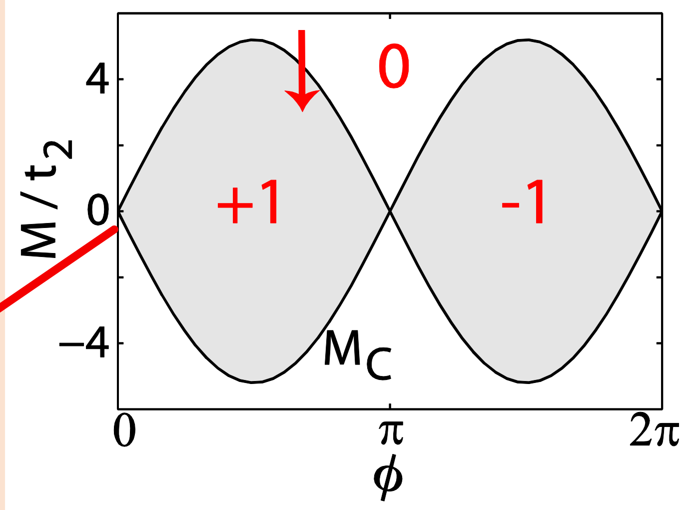
- **Haldane model** (infinite system)

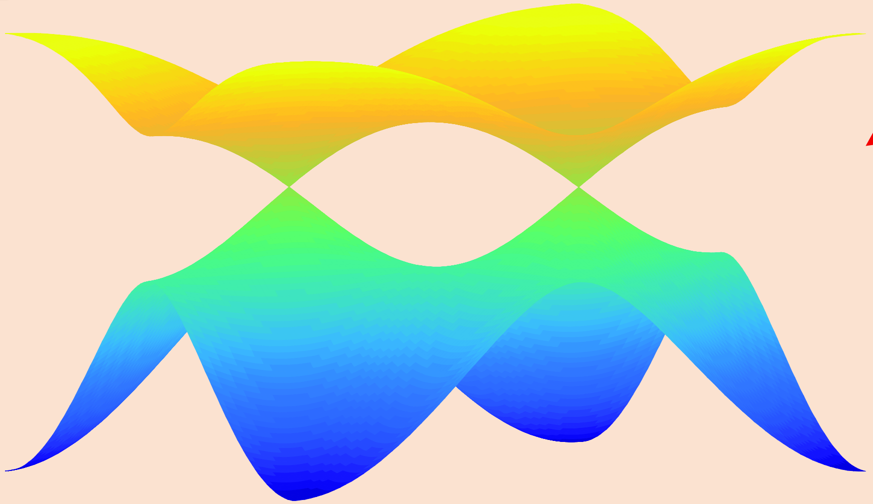
$$\mathcal{H} = -t_1 \sum_{\langle ij \rangle} a_i^\dagger a_j - t_2 \sum_{\langle\langle ij \rangle\rangle} e^{\pm i\phi} a_i^\dagger a_j + M \sum_A a_i^\dagger a_i - M \sum_B a_i^\dagger a_i$$



- Non-interacting fermions
- Non-uniform magnetic field, translational symmetry remains same
- 2 Dirac cones in the BZ
- ϕ breaks TRS
- M breaks IS

$M_C = \pm t_2 3\sqrt{3} \sin \emptyset\,(k)$





$t_2 = 0.1 t_1, \qquad \phi = 0.6\pi$

Theory: Haldane, PRL'88
Experiment: Jotzu et. al, Nature'14

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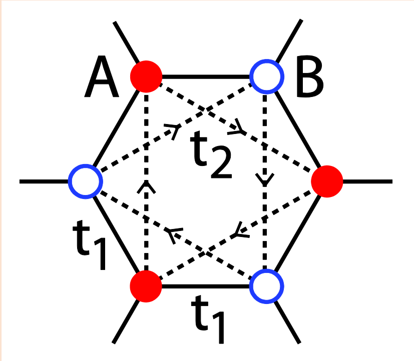

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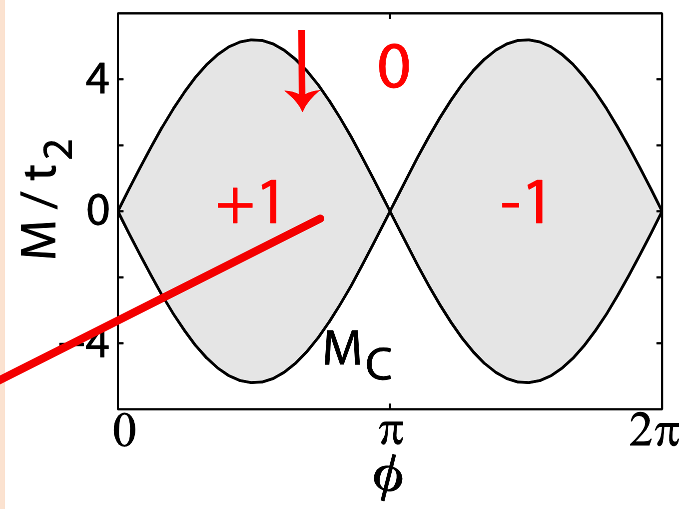

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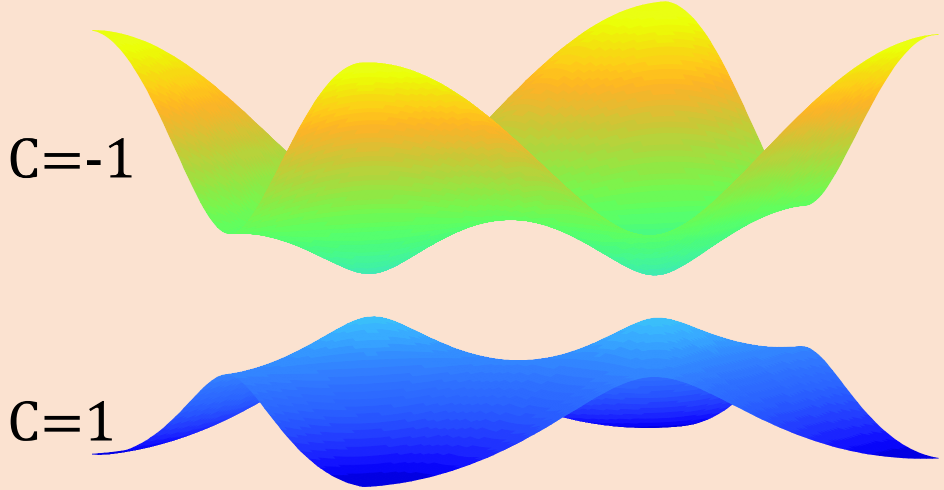

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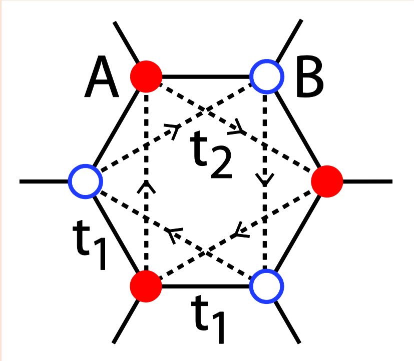

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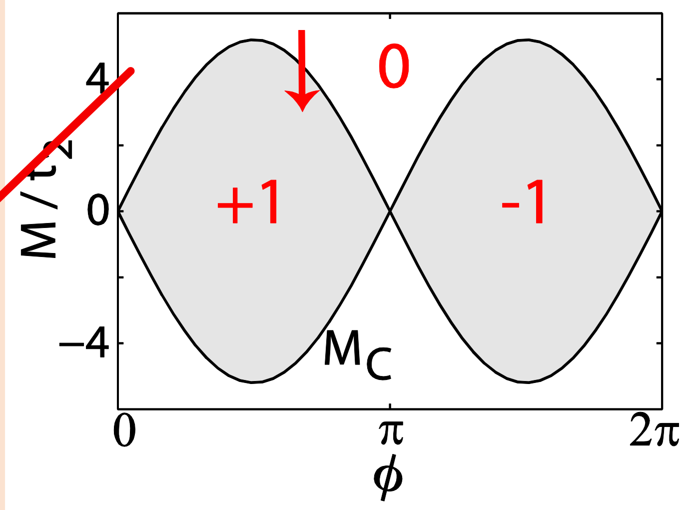

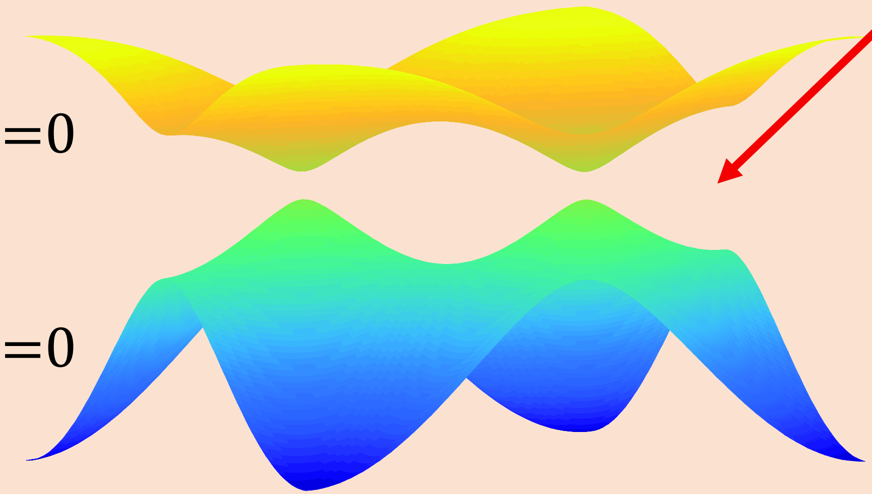

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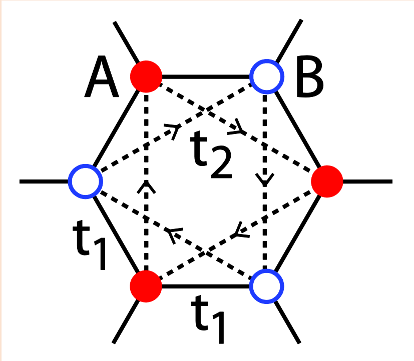

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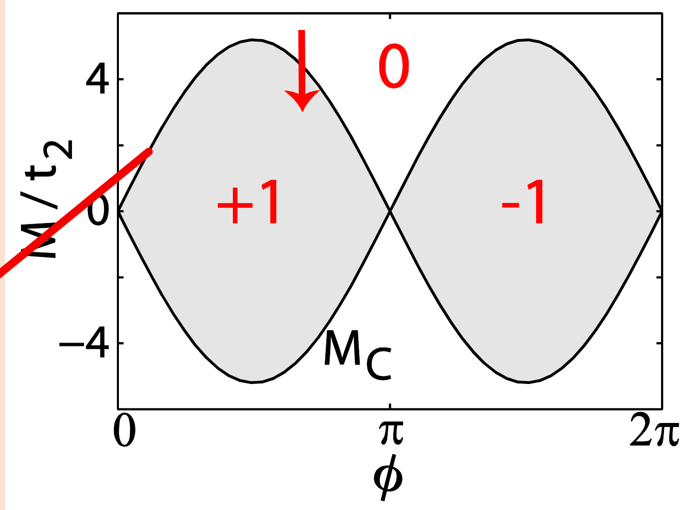

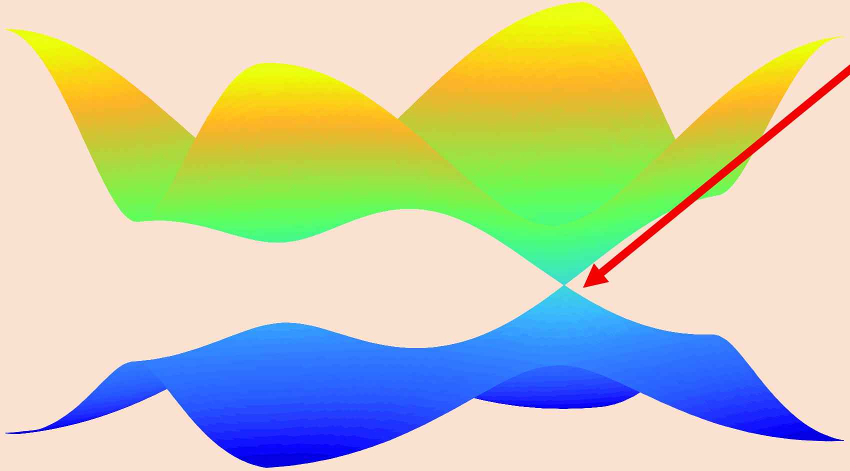

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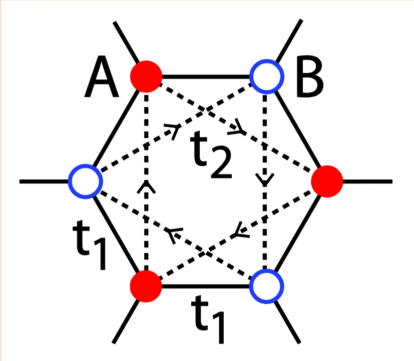
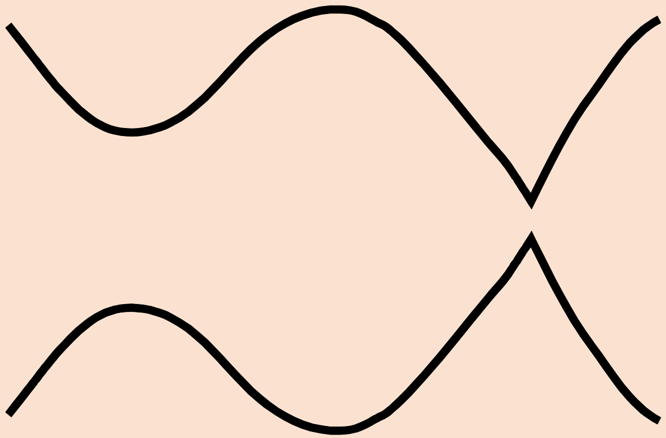


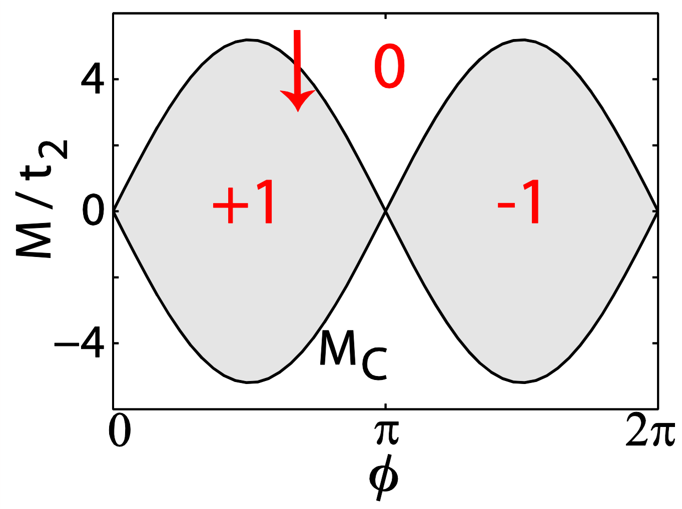
$M_C = \pm t_2 3\sqrt{3} \sin \emptyset\,(k)$

- Symmetric quenches around the transition point!

$M_i = M_C + \Delta M$

$M_f = M_C - \Delta M$





* Other quenches would also work

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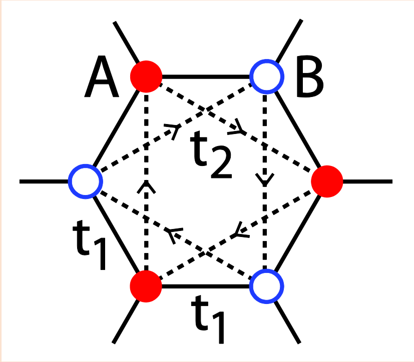

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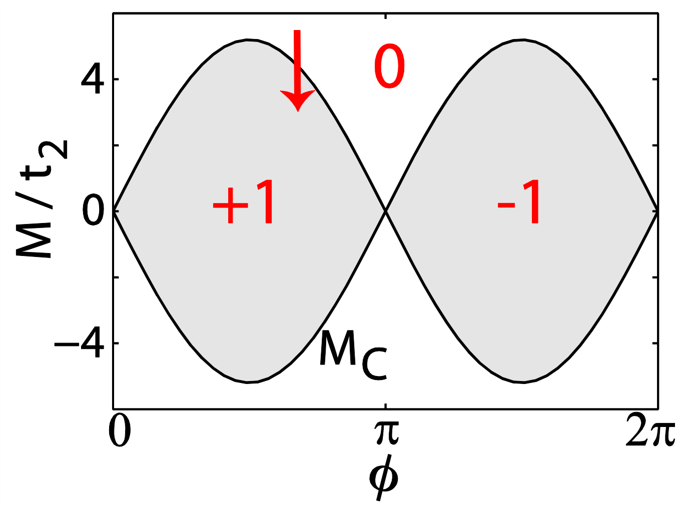

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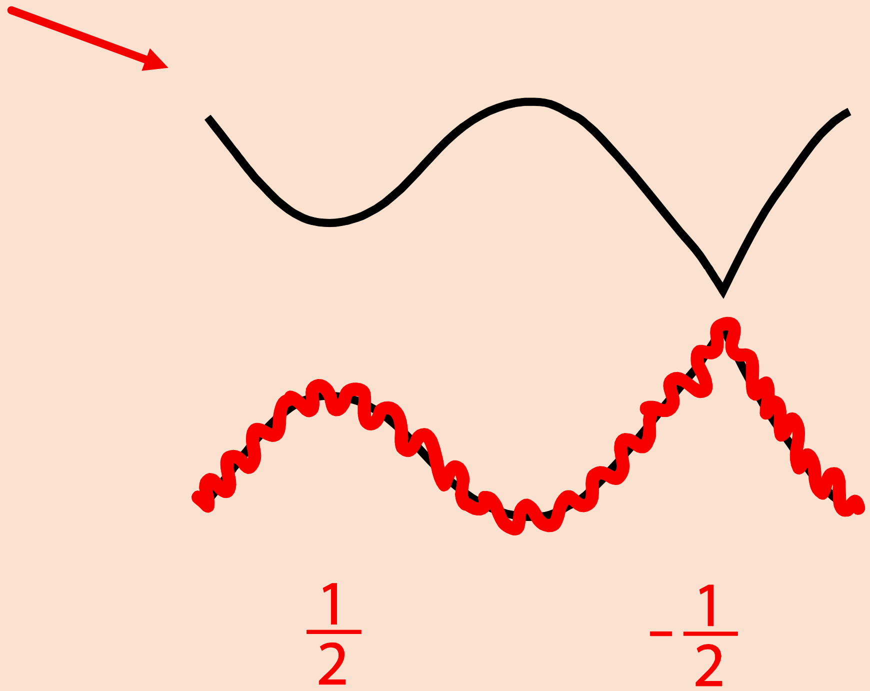

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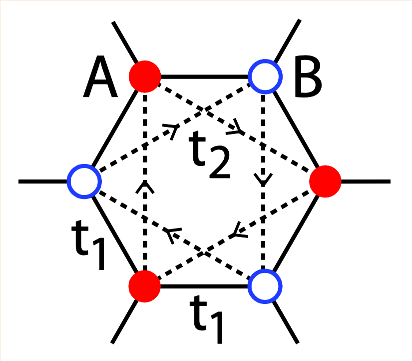
$$\mathcal{H} = -t_1 \sum_{\langle ij \rangle} a_i^\dagger a_j - t_2 \sum_{\langle\langle ij \rangle\rangle} e^{\pm i\phi} a_i^\dagger a_j + M \sum_A a_i^\dagger a_i - M \sum_B a_i^\dagger a_i$$

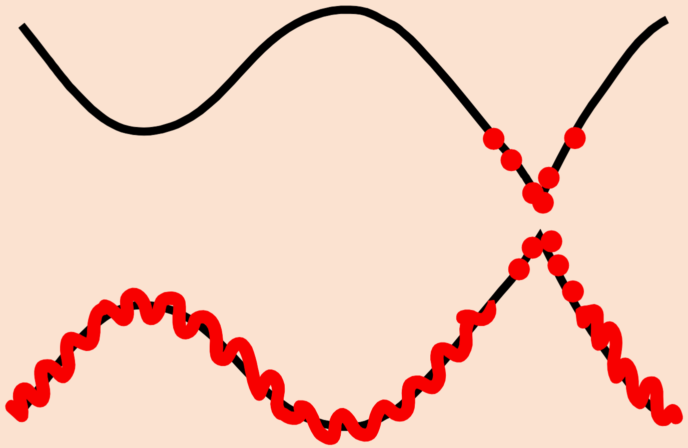


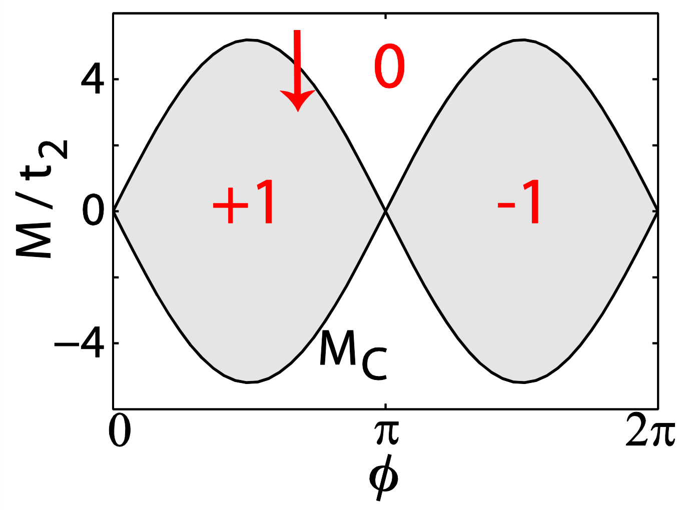
$M_C = \pm t_2 3\sqrt{3} \sin \emptyset \,(k)$

- Symmetric quenches around the transition point!

$M_i = M_C + \Delta M$

$M_f = M_C - \Delta M$





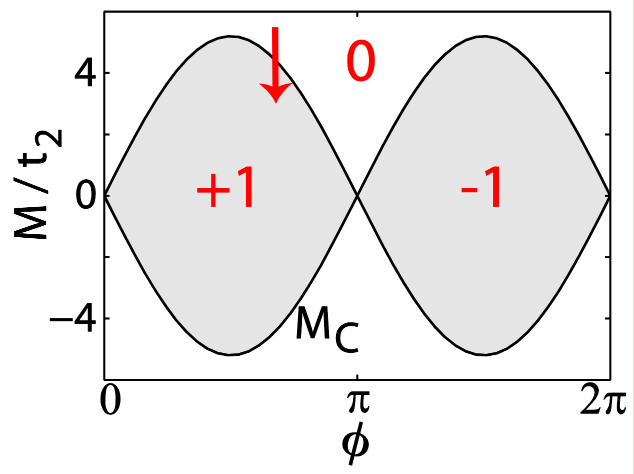
$t_2 = 0.1 t_1, \quad \phi = 0.6\pi$

$$\frac{1}{2} + \frac{1}{6} = \frac{2}{3} = \sigma_H^{neq}$$

- **Haldane model** (infinite system)

$M_i = M_C + \Delta M$

$M_f = M_C - \Delta M$



$M_C = \pm t_2 3\sqrt{3}\sin\emptyset\,(k)$

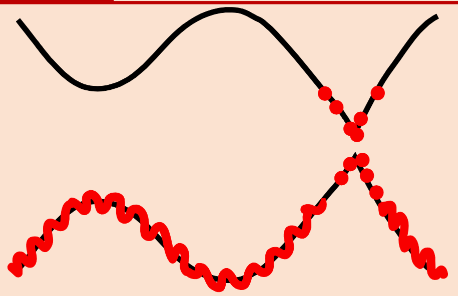
$t_2 = 0.1t_1, \qquad \phi = 0.6\pi$

- Numerically and Analytically
  - Quench
  - Calculate P_n at each k-point in the BZ
  - Calculate the Ω_n of the final bands
  - Hall conductivity $= C_{neq}$

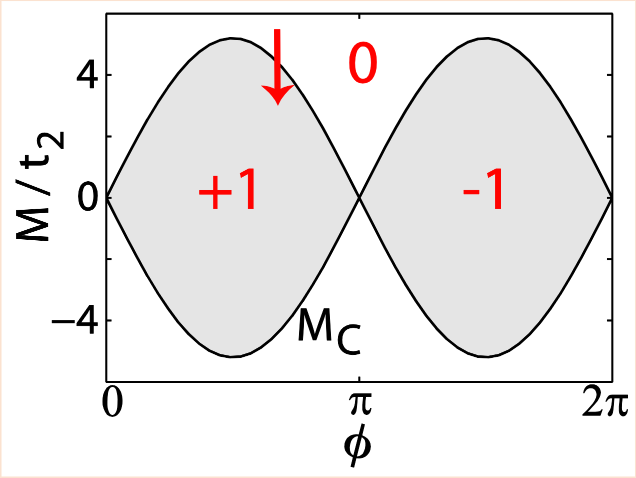
- **Haldane model** (infinite system)

$$M_i = M_C + \Delta M$$

$$M_f = M_C - \Delta M$$



$\frac{1}{2} + \frac{1}{6} = \frac{2}{3}$

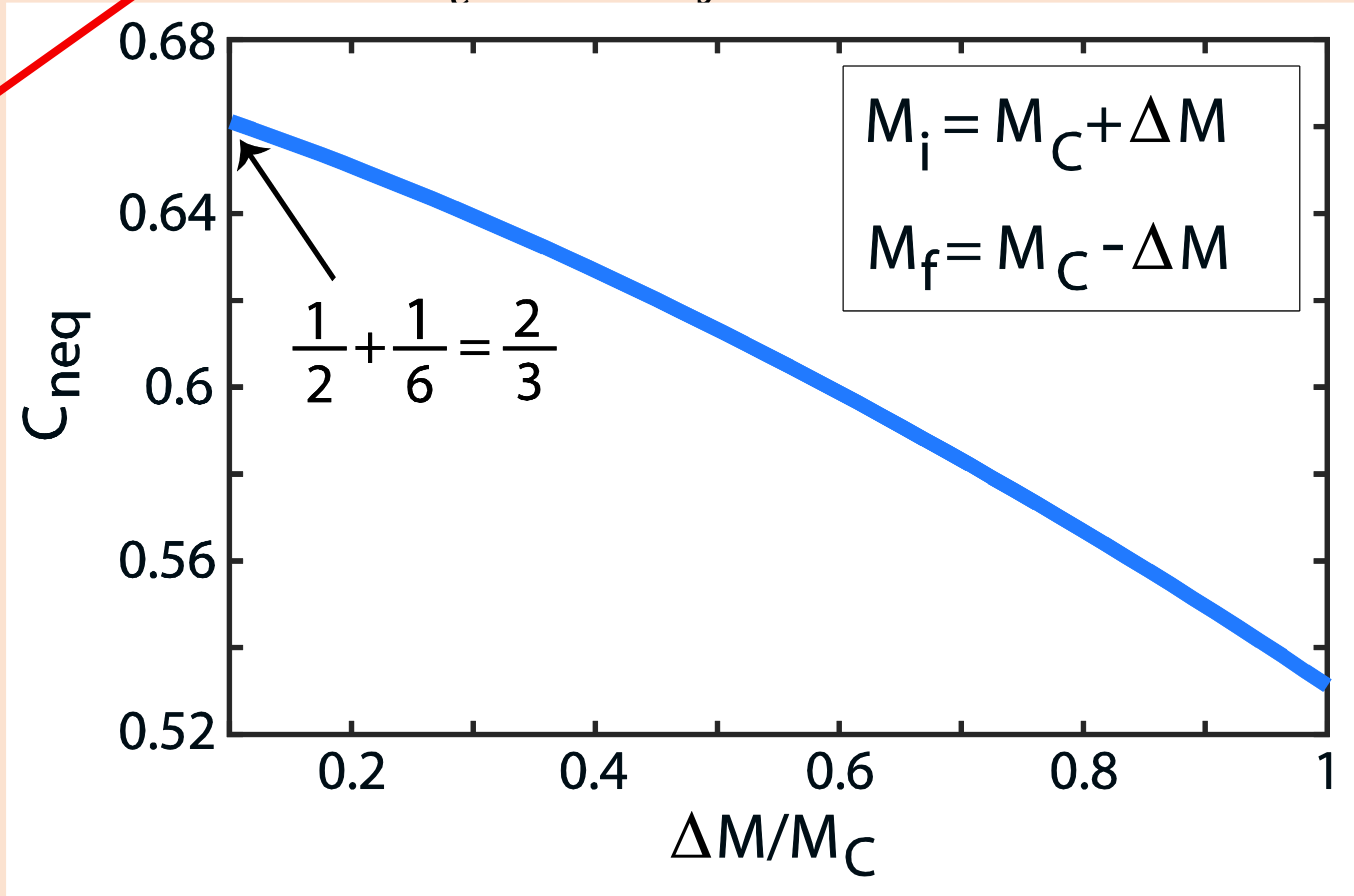


$$M_C = \pm t_2 3\sqrt{3} \sin \emptyset \, (k)$$

$$t_2 = 0.1 t_1, \qquad \phi = 0.6\pi$$

$\Delta M \ll M_c \implies$ only one Dirac cone

Actual # of particles excited is small

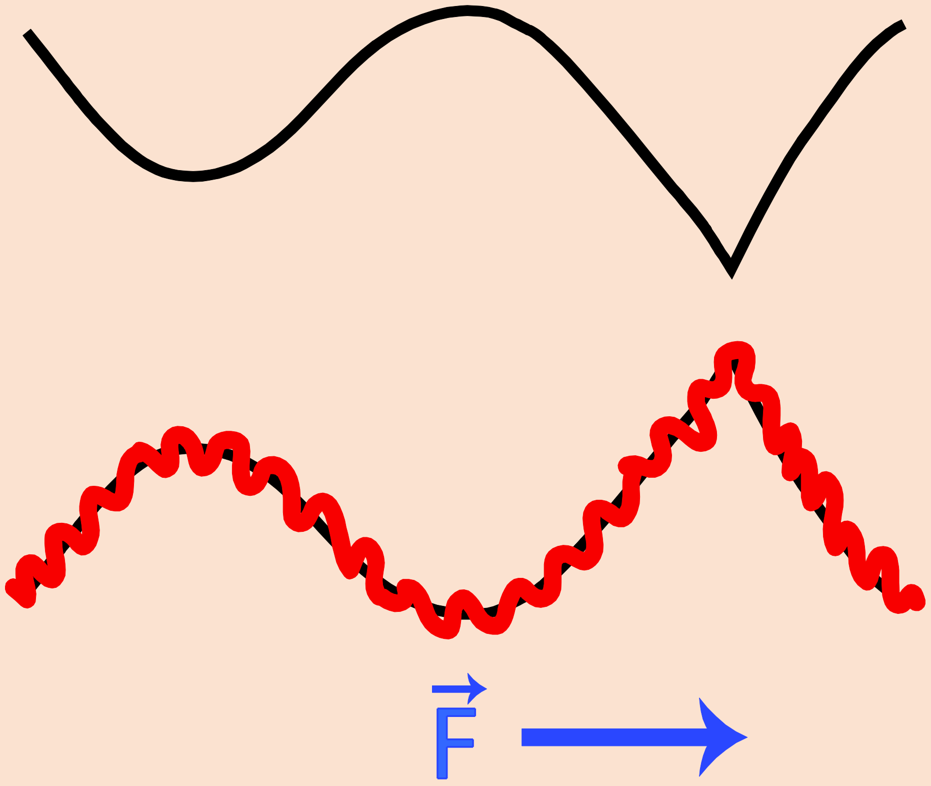


- Change in the Hall conductivity is again 2/3

- Non-equilibrium topological response can be different than equilibrium systems.
  - Linear response?
- Experimental considerations
  - Finite size?
  - Trap?

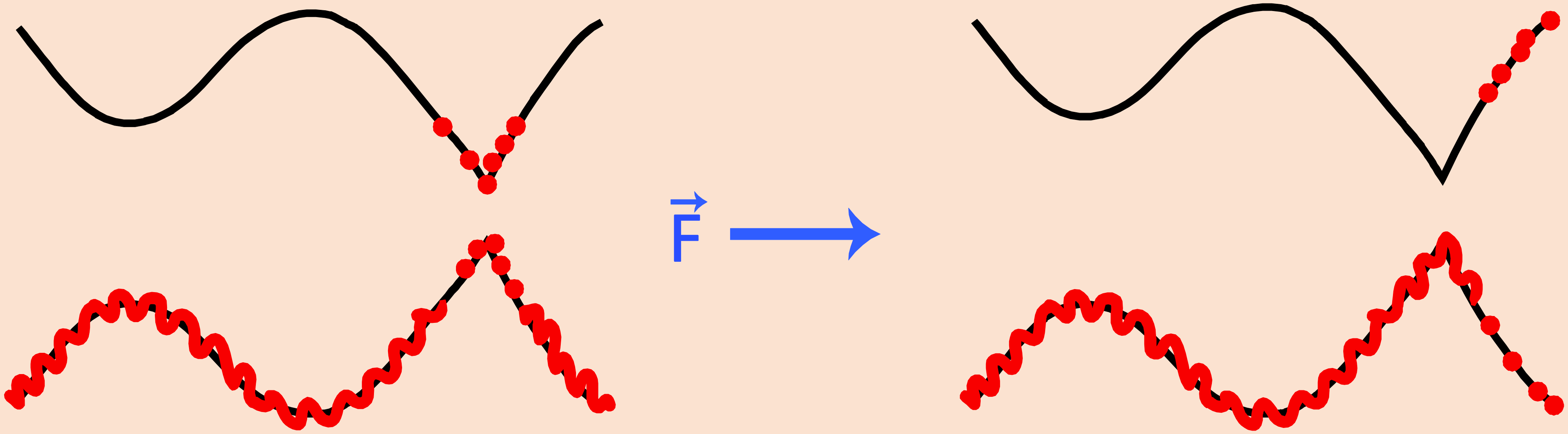
- (Beyond) Linear Response

- Equilibrium state: $\vec{F}$ applied => no change in occupation probabilities



- **(Beyond) Linear Response**

- Equilibrium state: $\vec{F}$ applied => no change in occupation probabilities
- Non-equilibrium: $\vec{F}$ applied => occupation probabilities shift



❖ Pronounced nonlinearities!

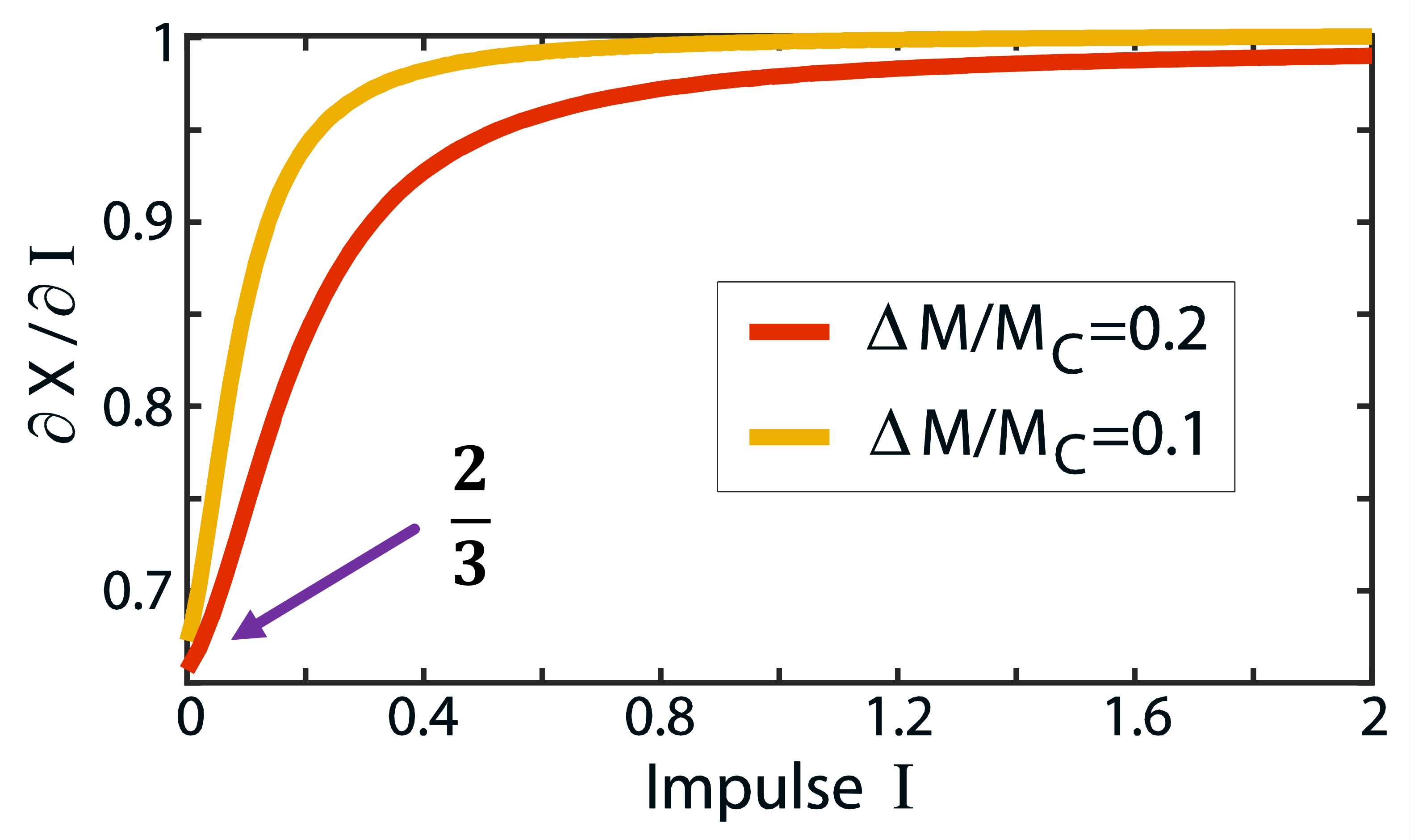
(Beyond) Linear Response

$$X(I) = \frac{1}{2\pi}\int_0^I dI' \sum_n \int_{Bz} d^2k\, P_n\left(\vec{k} - I'\hat{y}\right)\Omega_n\left(\vec{k}\right)$$

: Transverse displacement

Impulse $I = F\,\tau$

1. Quench $M_i = M_C + \Delta M$
   $M_f = M_C - \Delta M$
2. Apply impulse
3. Numerically calculate drift



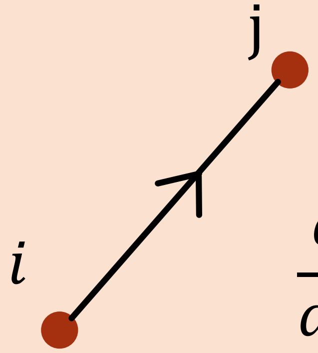
❖ Linear response valid when

$$I < \frac{\Delta}{c}$$

Take home message # 4

- In equilibrium: The bigger the impulse, the larger the transverse drift.
- Out-of-equilibrium: Strength of the impulse should be chosen carefully, to remain in the linear response regime.

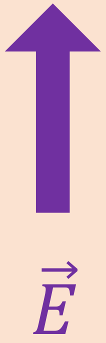
- **Haldane strip**

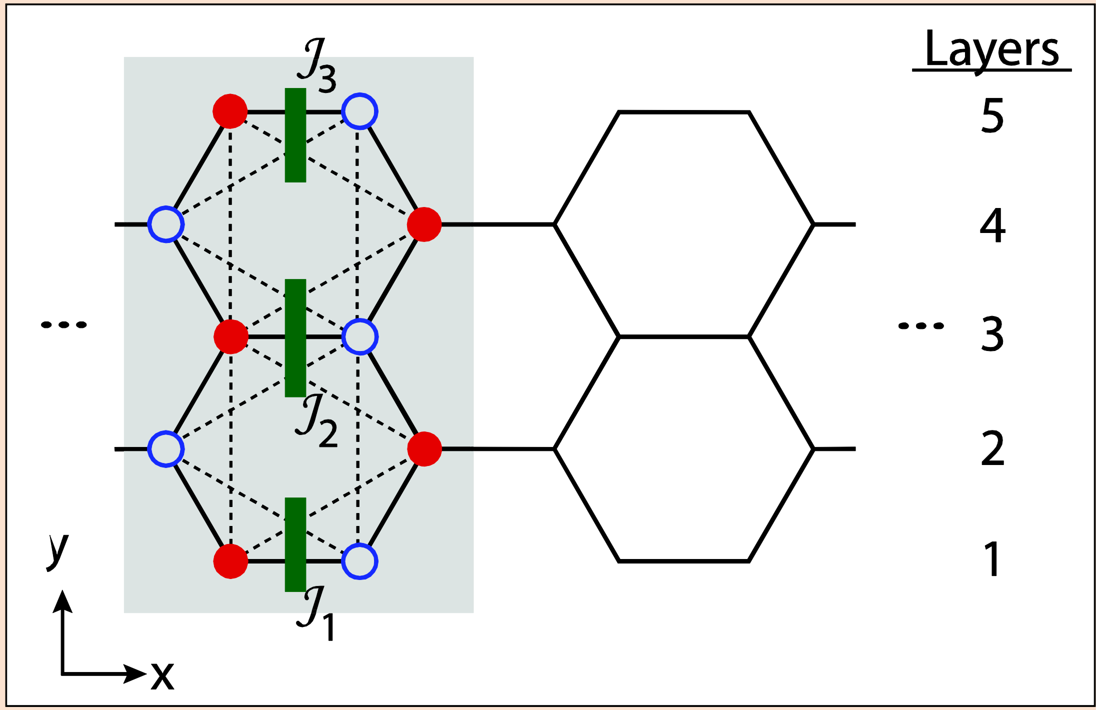


$$-t_{j\to i}\, a_i^{\dagger} a_j + h.c.$$

$$\frac{d}{dt}\left(a_i^{\dagger} a_i \right) = -\mathcal{J}_{i\to j}$$

$$< \mathcal{J}_{i\to j} >= 2\,\mathrm{Im}\left\{ t_{j\to i} < a_i^{\dagger}\, a_j > \right\}$$

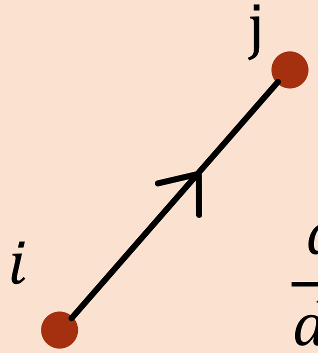




- OI => $J_H = \sum \mathcal{J}_\ell$ is always zero, nonzero currents in individual links
- CI => J_H is finite for $\vec{E} \neq 0$
- Initial ground state, quench, apply small electric field

$$\sigma_H = \frac{1}{W}\frac{\partial J_H}{\partial E}$$

- **Haldane strip**



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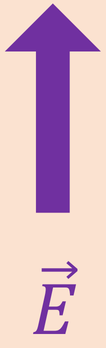
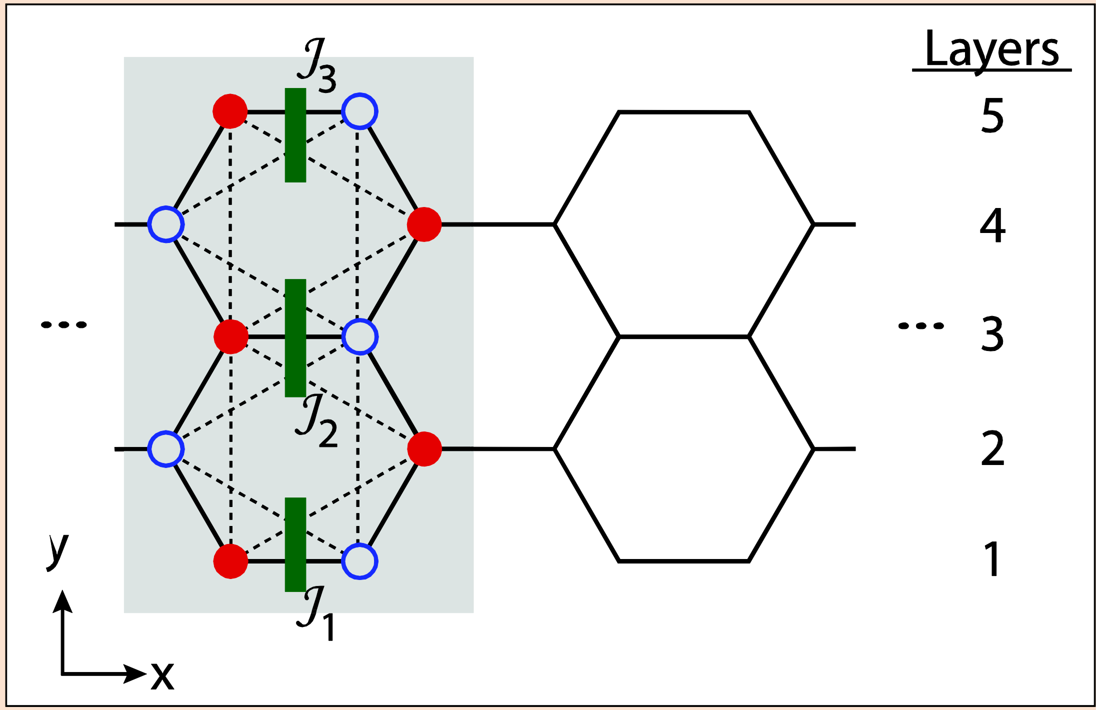




- OI => $J_H = \sum \mathcal{J}_\ell$ is always zero, nonzero currents in individual links
- CI => J_H is finite for $\vec{E} \neq 0$
- Initial ground state, quench, apply small electric field, numerically calculate currents

$$\sigma_H = \frac{1}{W}\frac{\partial J_H}{\partial E}$$

Small symmetric quench, $\frac{2}{3}$ ✓.

Experiments

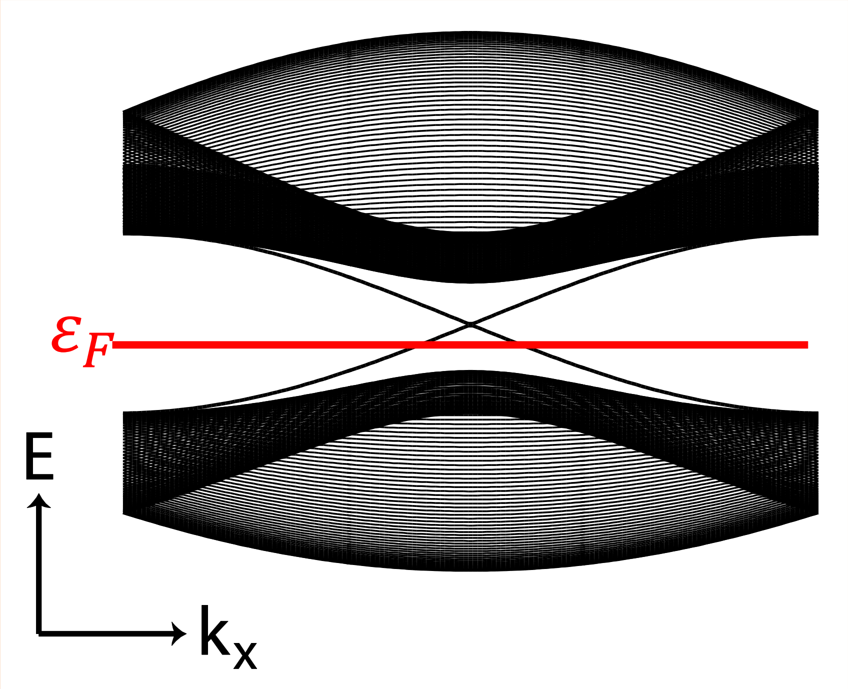
- Linear response regime
- Specifically in cold atoms
  - Finite size ?
  - Trap?

→ Even in the ground state

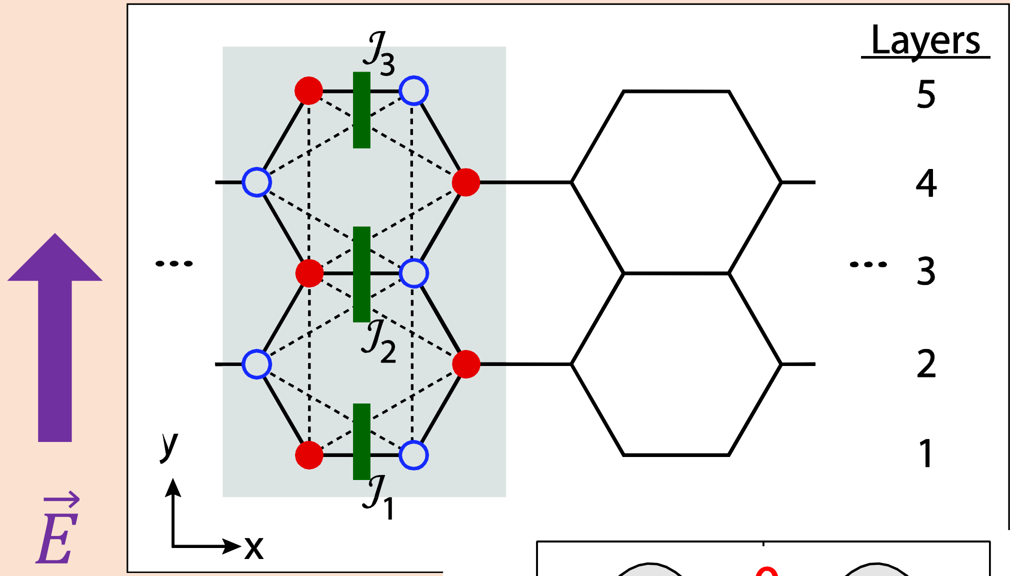
- **Finite size effects**

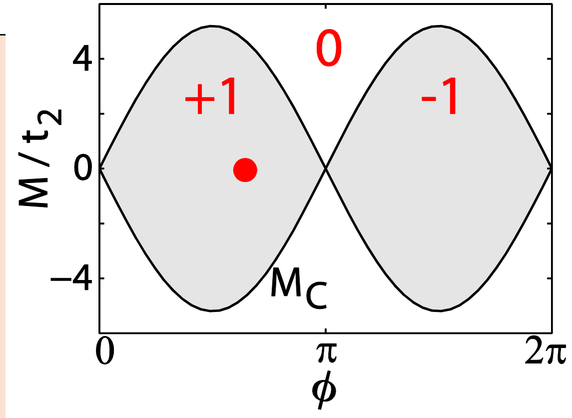
- Cold atoms can explore strips of varying width

$t_2 = 0.1t_1, \phi = 0.6\pi, M = 0,$ deep in the CI regime



L= 101 layers



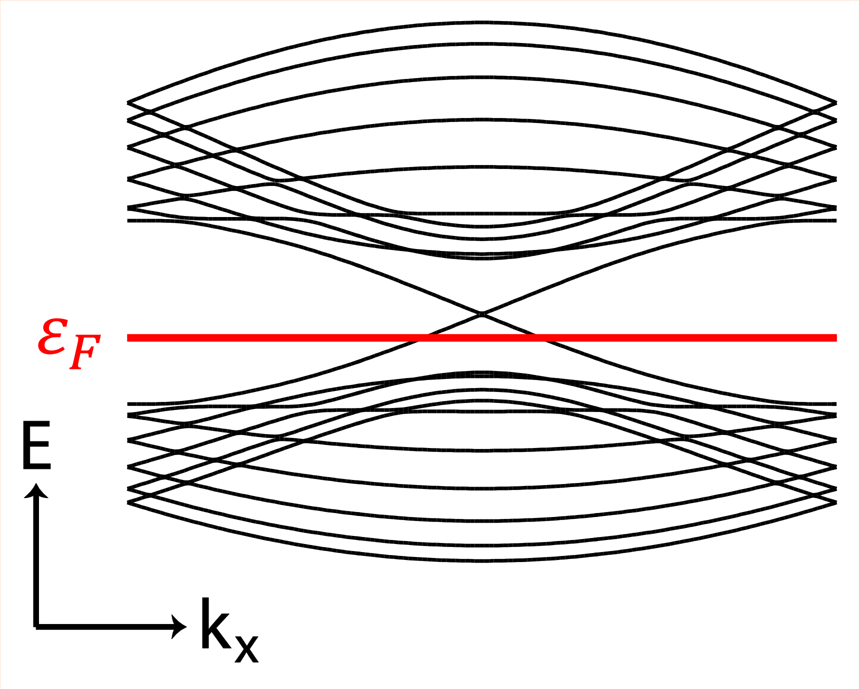


Infinite system calculations predict C=1

- **Finite size effects**

- Cold atoms can explore strips of varying width

$t_2 = 0.1t_1, \phi = 0.6\pi, M = 0,$ deep in the CI regime



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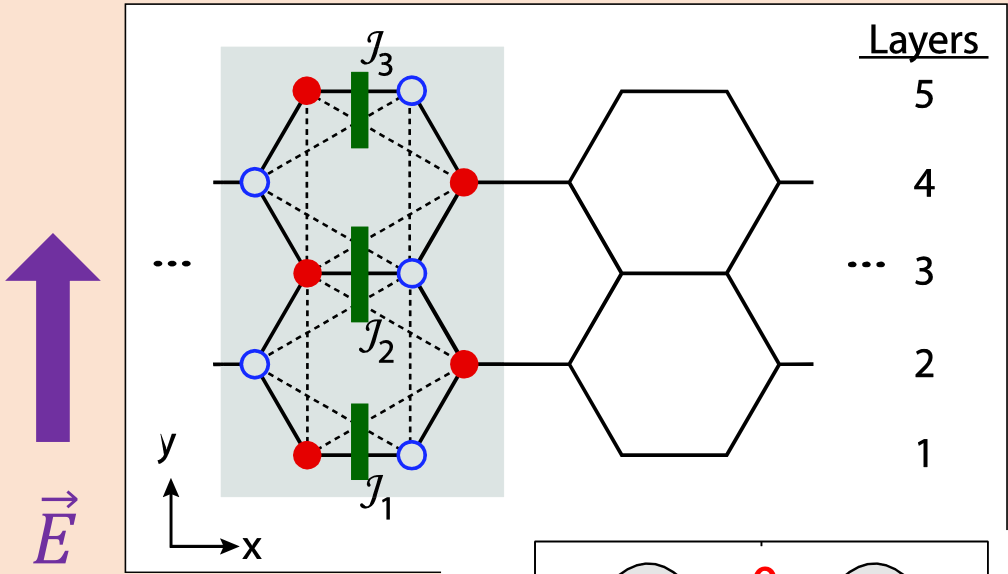

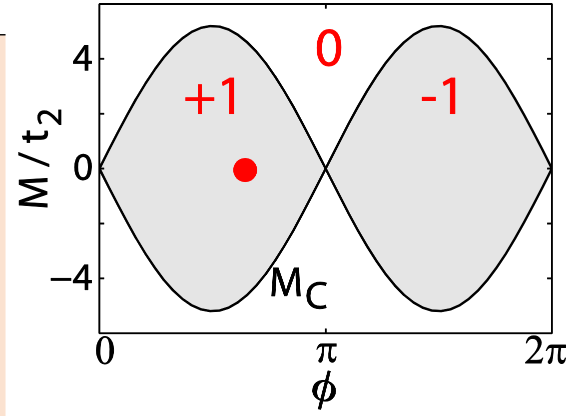

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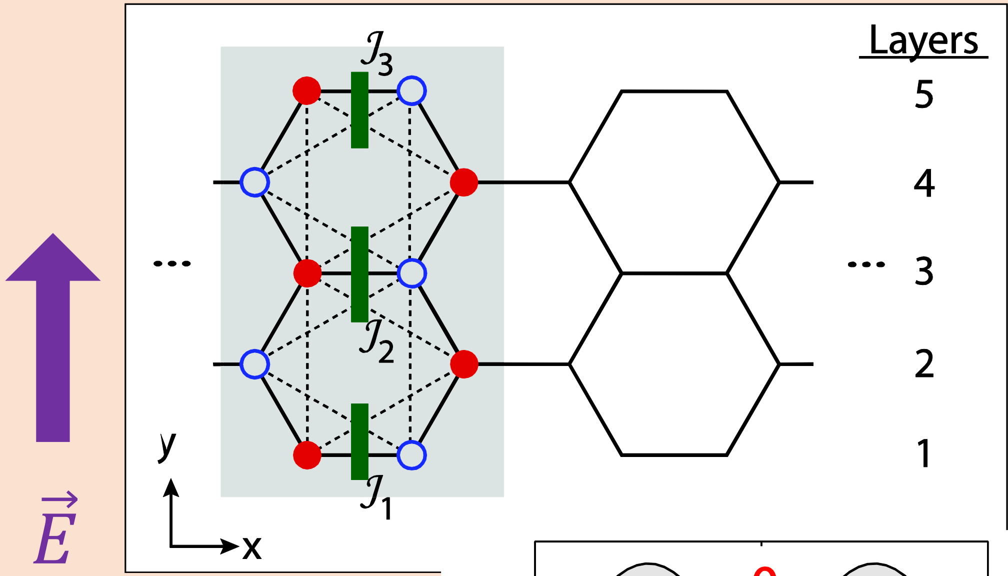
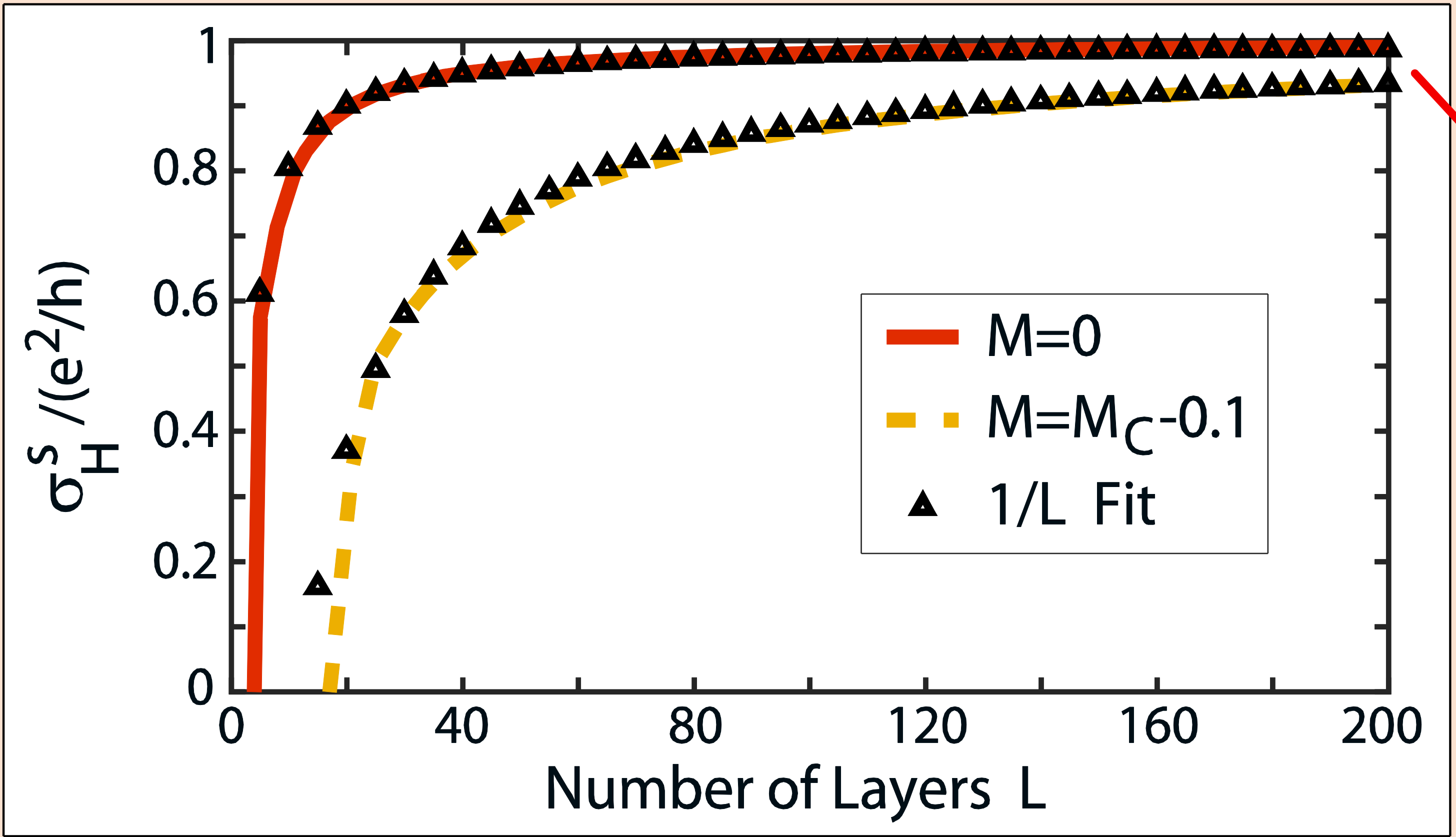
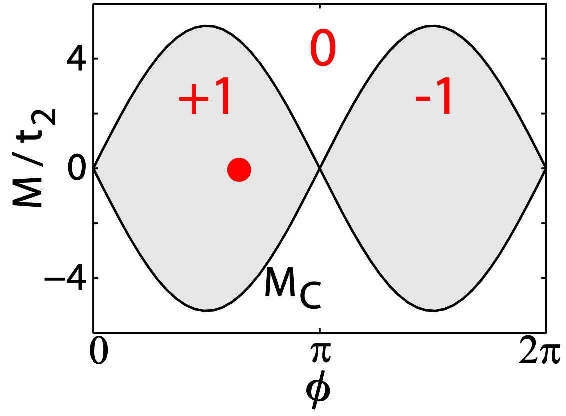




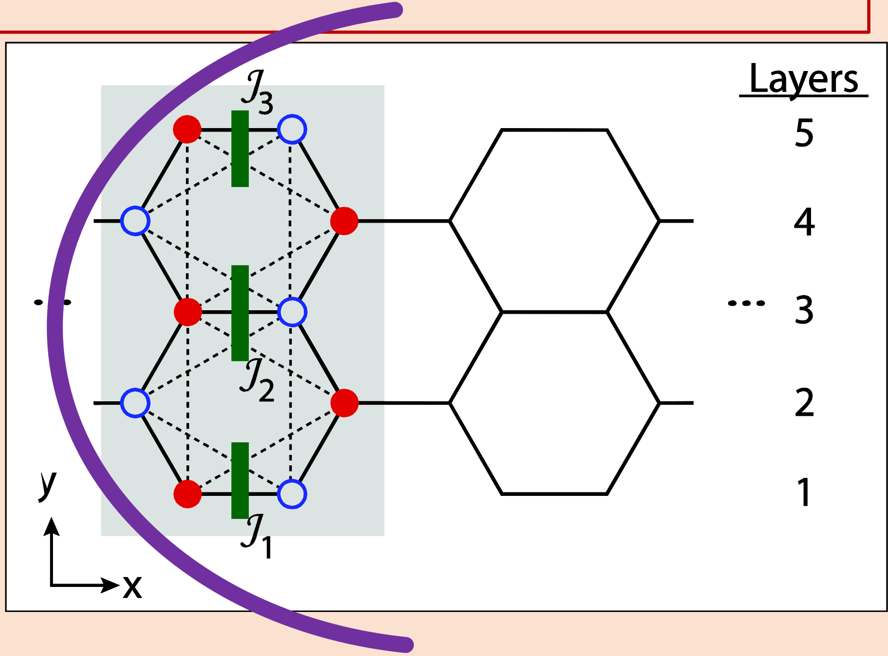
Infinite system result

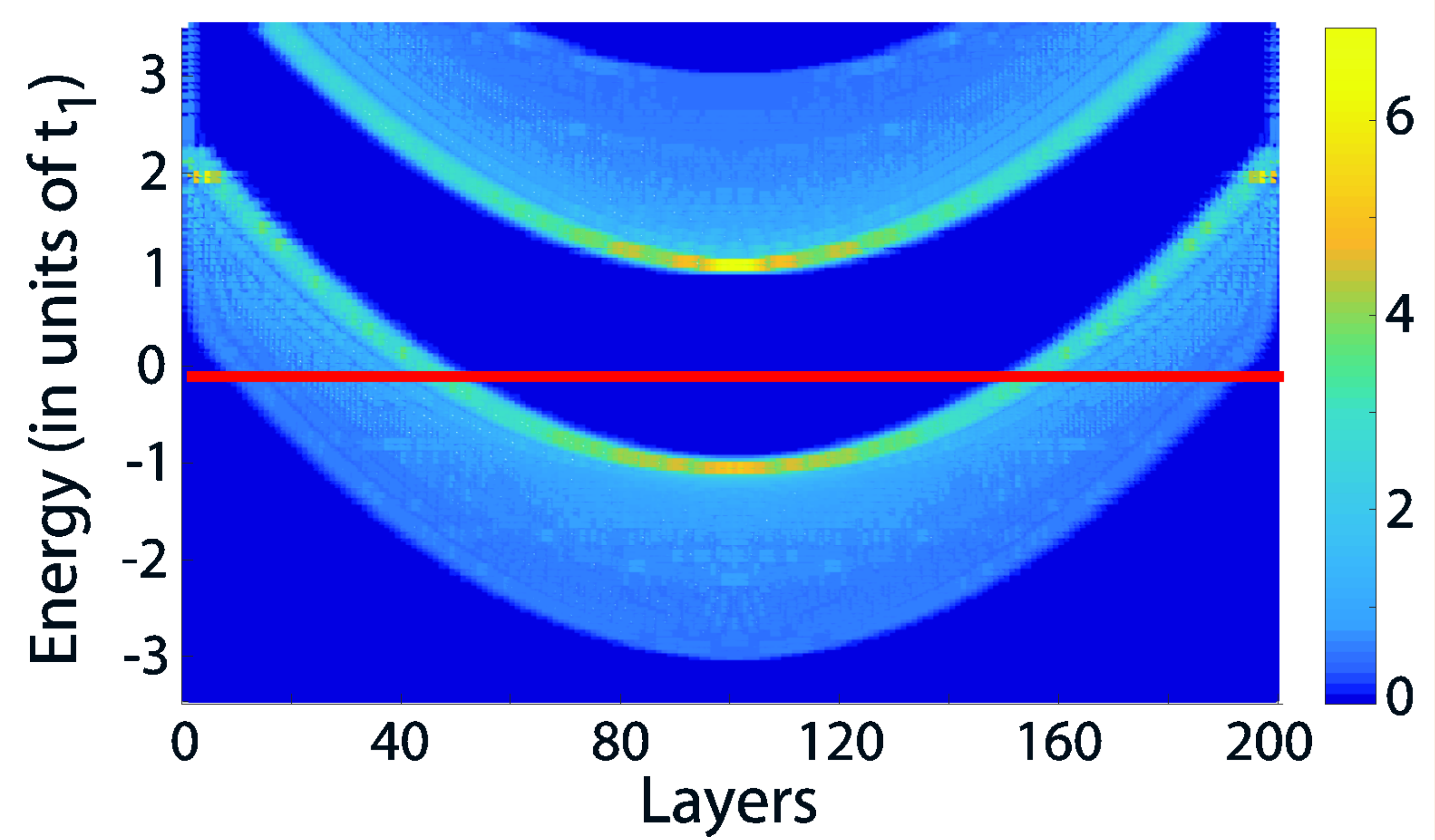


$$\frac{\#\text{ of edge modes}}{\#\textit{ of bulk modes}} \sim \frac{2}{L-2}$$

- **Trap**

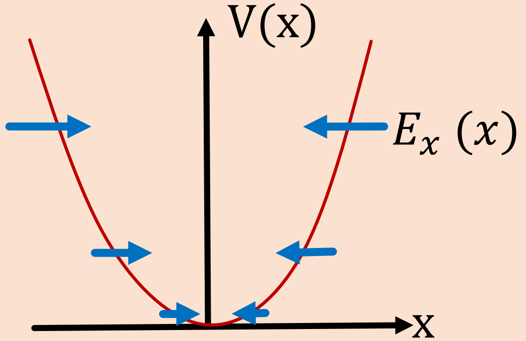
$$LDOS(y,\varepsilon) = \sum_n |\Psi^n(y)|^2 \Theta\left(\varepsilon + \frac{\delta\varepsilon}{2} - \varepsilon_n\right) \Theta\left(\varepsilon_n - \varepsilon - \frac{\delta\varepsilon}{2}\right)$$

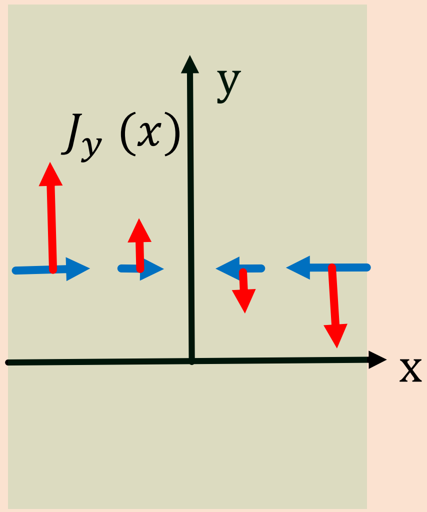




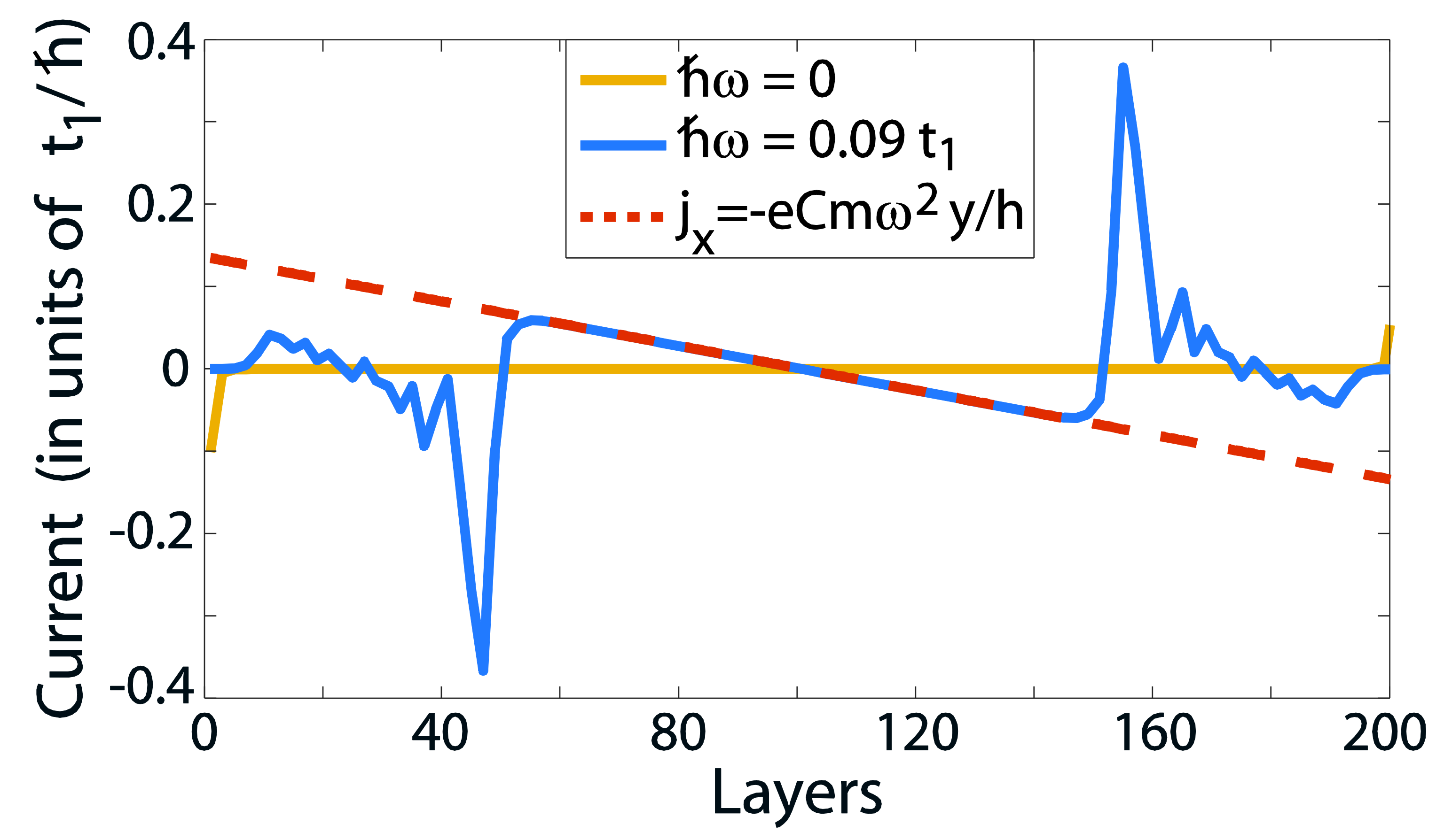
- Edges are always metallic
- Washes out any potential signal from topological modes
- Yields non-universal results…

- **Trap**





In the absence of an electric field!



Net current vanishes, but

- *Quantized anomalous Hall currents*

$$E_y^{trap} = \frac{-m\omega^2 y}{e}$$

$$J_x = \sigma_H E_y^{trap} = -\frac{e}{h} C m\omega^2 y$$

- In general, σ_H^{neq} is non-universal, not given by the Chern number.
- In mass-sign-inverting quenches, universal fractional Hall response.

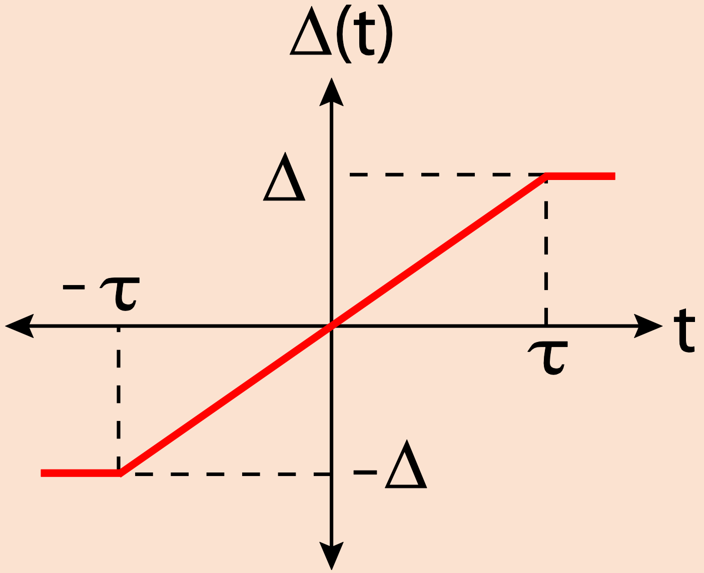
$$\Delta \rightarrow -\Delta \quad \Rightarrow \quad \sigma_H^{neq} = \frac{1}{6}$$

- Can be seen in small quenches of any topological lattice system, e.g. Haldane model
- Out-of-equilibrium Hall response linear regime is small.
- Hall response of a strip saturates to the infinite system result with a power law for increasing strip width.
- In a harmonic trap, local currents can reveal the topology of the system even in the absence of an external electric field.

Nur Ünal, Erich Mueller, MOO, PRA **94**, 053604 (2016)
arXiv:1608.04395

Thank you for your attention!

Linear Ramp



$$i\hbar \frac{\partial}{\partial t}\left|\Psi\left(\vec{k},t\right)\right\rangle = H\left(\vec{k},t\right)\left|\Psi\left(\vec{k},t\right)\right\rangle$$

- At each k, independent LZ problem

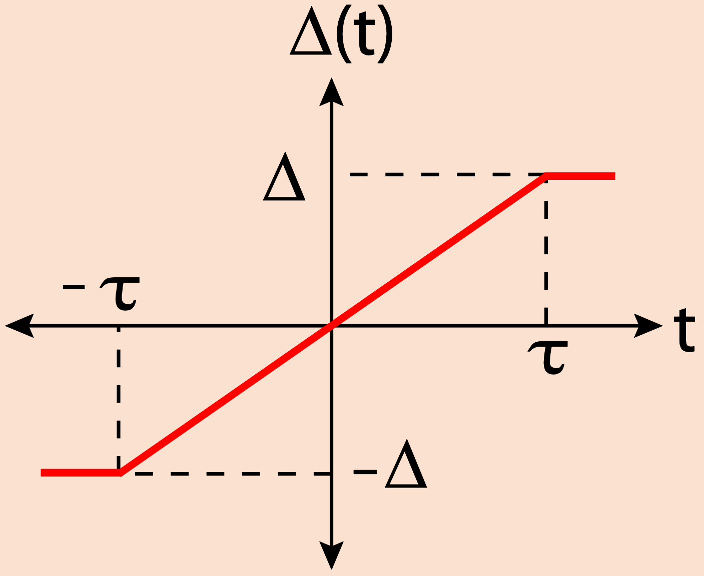
$$\eta_k = \frac{c^2 k^2 \tau}{\hbar\Delta} \gg 1 \quad => \text{ Remain in lower band}$$

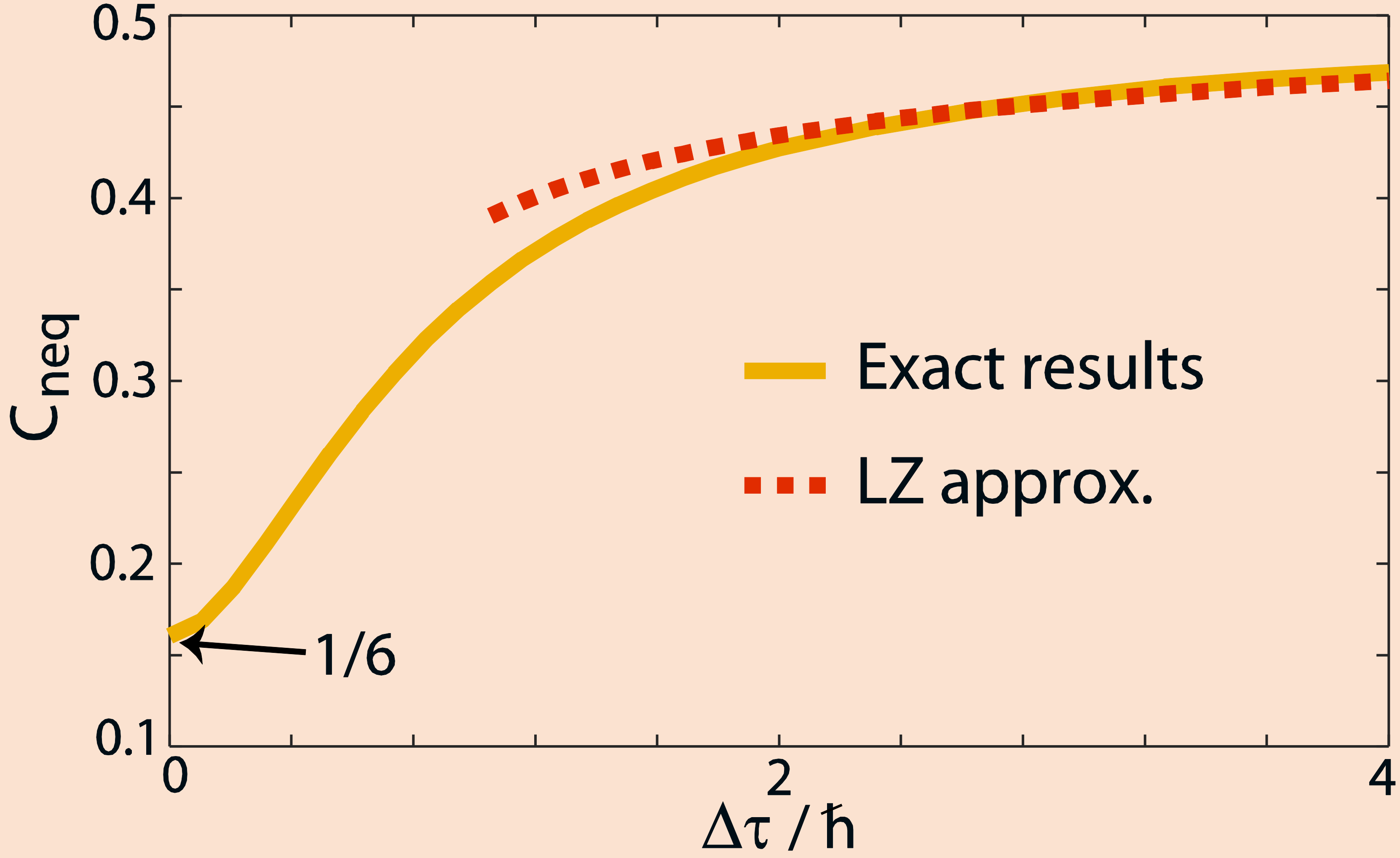
$$\ll 1 \quad => \text{ Transition}$$

- Adiabaticity always holds for large k
- If the gap is large, $\Delta \gg \hbar/\tau$, $\qquad P_2\left(\vec{k},\tau\right) = e^{-\pi c^2 k^2 \tau/\hbar\Delta}$

$$C_{neq}^{LZ} = -\frac{1}{2} + \pi\sqrt{\frac{\Delta\tau}{\hbar}}\, e^{\pi\Delta\tau/\hbar} erfc\left(\sqrt{\pi\frac{\Delta\tau}{\hbar}}\right)$$

Linear Ramp





- What happens when a system is suddenly driven between two topologically different phases?
  - Chern number is preserved!

$$C = \frac{1}{2\pi}\int dk_\mu\, dk_\nu\, \Omega_{\mu\nu} = \cdots |a(\infty)||b(\infty)|\ \cos[t\ (\varepsilon_\ell(k=\infty) - \varepsilon_u(k=\infty))]$$

Zero, unless something is coming in…

$$\Omega_{\mu\nu} = \partial_{k_\mu} A_{k_\nu} - \partial_{k_\nu} A_{k_\mu}$$

$$A_{k_\mu} = i\left\langle \Psi \middle| \partial_{k_\mu} \middle| \Psi \right\rangle$$

- Two-band model, e.g. Dirac Hamiltonian
  - Initially filled lower band, excite some particles to upper band

$$\Psi(k,\theta,t) = a(k,t)\,|\ell(k,\theta)\rangle\ +\ b(k,t)\,|u(k,\theta)\rangle$$

eigenstates of the final Hamiltonian

Bhaseen, PRL'15

Harmonic Trap = Non-Uniform Hall Bar