Kuantum $\frac{1}{\sqrt{2}}\left(|\text{Optiği}\rangle \overset{+}{\text{ve}} |\text{Bilisim}\rangle\right)$ Toplantısı

Many-particle entanglement criterion for superradiant-like states

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Outline

- Relation among
  i. single-mode nonclassicality
  ii. two-mode entanglement
  iii. many-particle entanglement

- new N-particle entanglement criterion
  i. test for Dicke states
  ii. test for the ground state Dicke Hamilonian (superradiance)
  iii. test for single-photon superradiance (exact, time depndt)
  iv. test for random superposition of Dicke states
  v. ground state of an interacting BEC

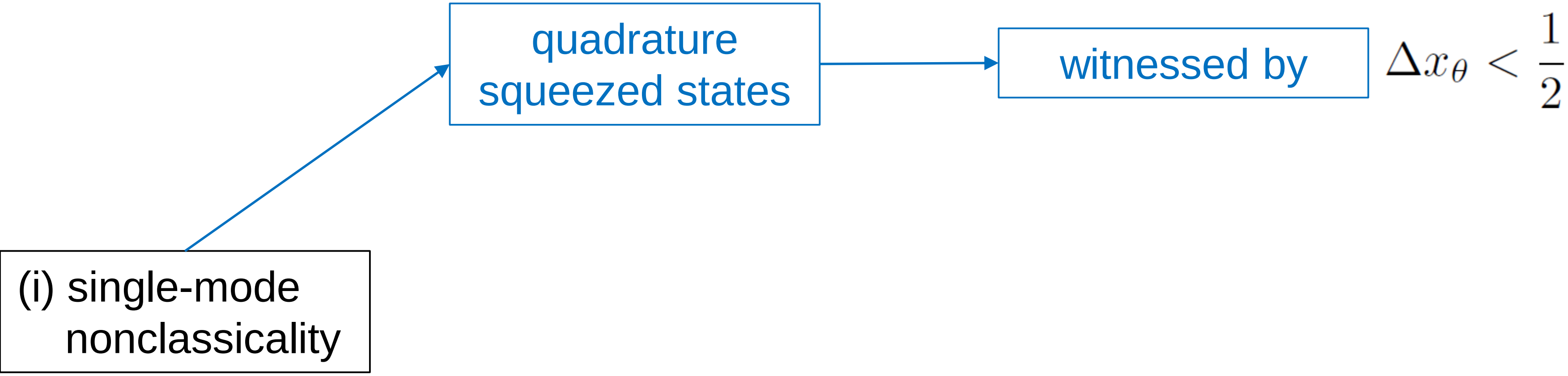
3 kind of nonclassicalities

(i) single-mode nonclassicality

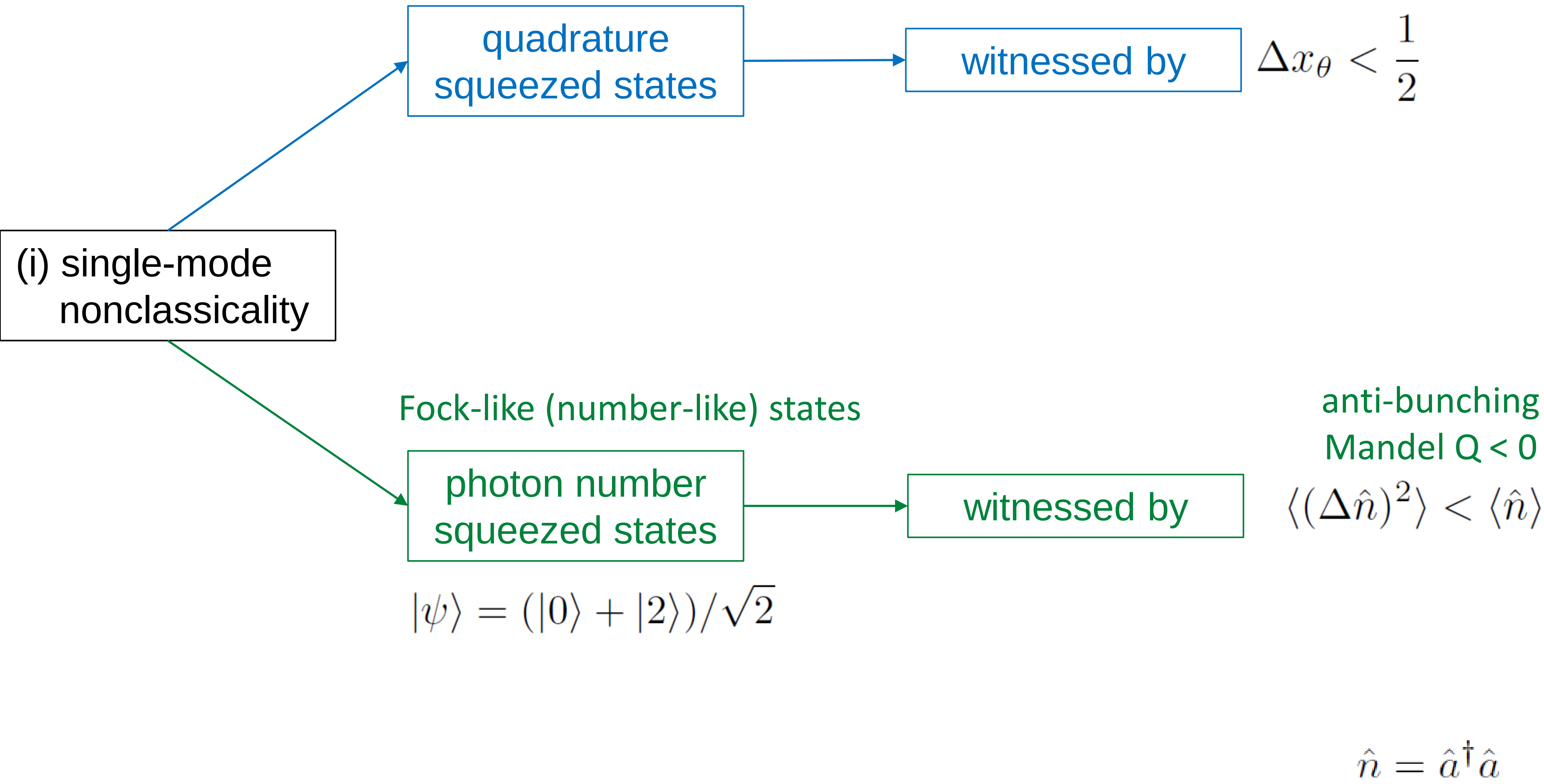
(ii) two-mode entanglement

(iii) many-particle inseperability

(i) single-mode nonclassicality



(i) single-mode nonclassicality



(i) single-mode nonclassicality

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quadrature squeezed states → witnessed by $\Delta x_\theta < \frac{1}{2}$

Fock-like (number-like) states

photon number squeezed states → witnessed by

anti-bunching
Mandel Q < 0

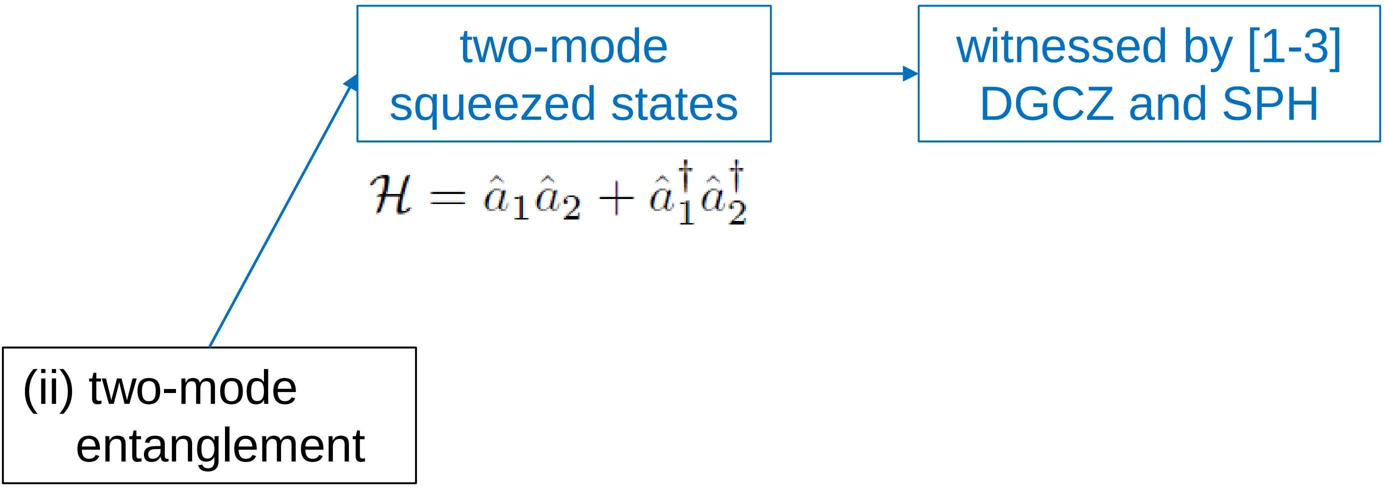
$\langle (\Delta \hat{n})^2 \rangle < \langle \hat{n} \rangle$

works fine for superposition of Fock-states

$|\psi\rangle = (|0\rangle + |2\rangle)/\sqrt{2}$

$\hat{n} = \hat{a}^\dagger \hat{a}$

(ii) two-mode entanglement



$$\mathcal{H} = \hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger$$

$$\langle (\Delta \hat{u})^2 \rangle \langle (\Delta \hat{v})^2 \rangle < 1$$

$$\hat{u} = x_1 + x_2$$

$$\hat{v} = p_1 - p_2$$

[1] Lu-Ming Duan, G. Giedke, J. I. Cirac, and P. Zoller, Phys. Rev. Lett. **84**, 2722 (2000).
[2] R. Simon, Phys. Rev. Lett. **84**, 2726 (2000).
[3] Stefano Mancini, Vittorio Giovannetti, David Vitali, and Paolo Tombesi, Phys. Rev. Lett. **88**, 120401 (2002).
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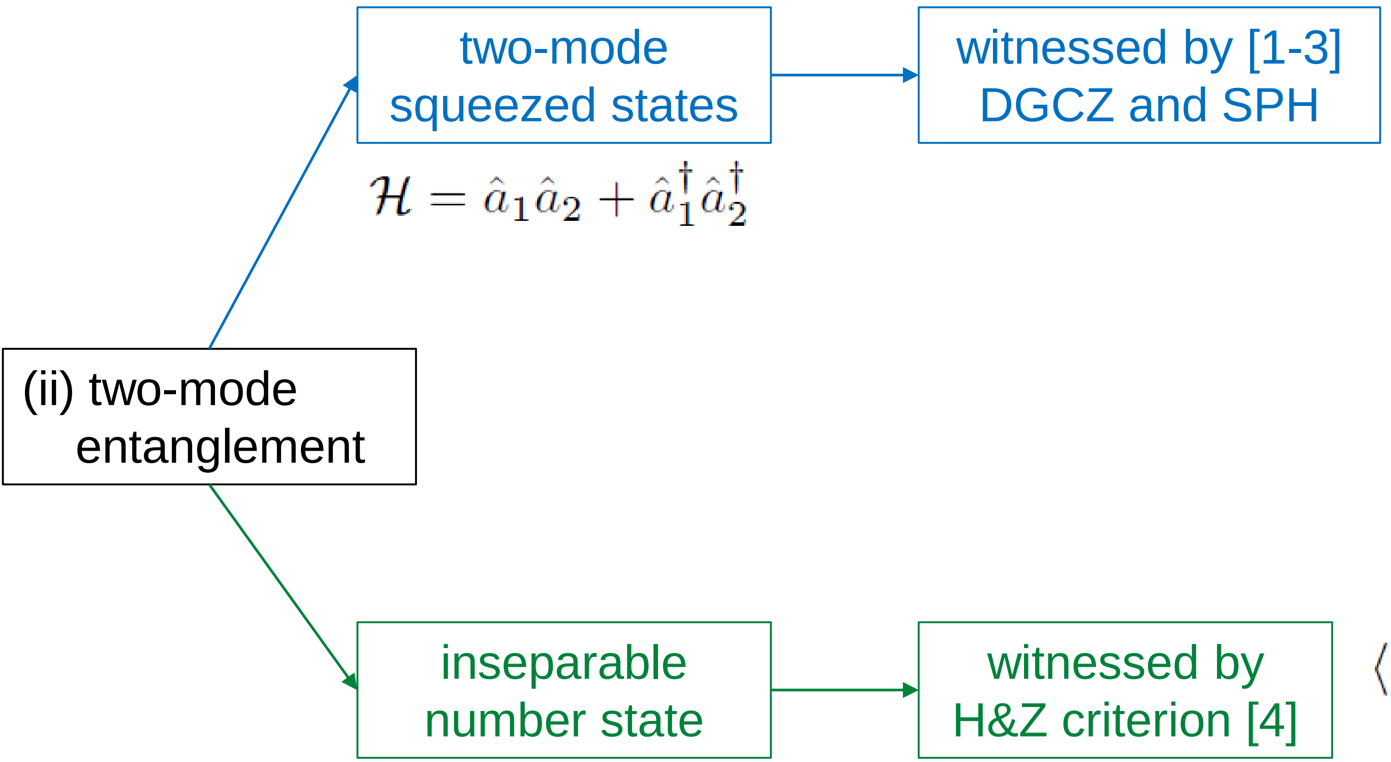

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$$\hat{v} = p_1 - p_2$$

$$\langle\hat{a}_1^\dagger\hat{a}_1\hat{a}_2^\dagger\hat{a}_2\rangle < |\langle\hat{a}_1^\dagger\hat{a}_2\rangle|^2$$

works fine for superposition of number-states

$$|\psi\rangle = (|3\rangle|1\rangle + |2\rangle|2\rangle)/\sqrt{2}$$

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(iii) many-particle inseparability

spin-squeezed states → witnessed by [7]

$$\xi = \frac{N\langle(\Delta S_z)^2\rangle}{\langle S_x\rangle^2 + \langle S_y\rangle^2} < 1$$ [7]

$\mathcal{H} = S_+^2 + S_-^2$ $\quad$ $\mathcal{H} = S_z^2$ [5]

$\mathcal{H} = S_+ J_+ + S_- J_-$ [6]

(iii) many-particle inspeperability

[5] Kitagawa, M. & Ueda, M. Squeezed spin states. Phys. Rev. A **47**, 5138 (1993).

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(iii) many-particle inspeperability

Dicke states (number-like)

Duan **[8]** ✓

superpositions of Dicke states ??

eg. superradiant states?

Duan **[8]** ✗

ξ_{new} **? ? ?**

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a question in place

Atomic coherent states (ACS)

$$|\xi_{\rm ACS}\rangle = (|g\rangle_1 + \xi|e\rangle_1) \otimes (|g\rangle_2 + \xi|e\rangle_2) \ldots \otimes (|g\rangle_N + \xi|e\rangle_N)$$

separable many-particle states

$$\mathcal{H}_1 = S_+J_+ + S_-J_- \xrightarrow{\text{operate}} |\xi_{\rm ACS}\rangle$$

⇨ generates **many-particle** entanglement

$$\mathcal{H}_2 = S_+J_- + S_-J_+ \xrightarrow{\text{operate}} |\xi_{\rm ACS}\rangle$$

⇨ cannot generate entanglement

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$$\mathcal{H}_1 = \hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger \xrightarrow{\text{operate}} |\alpha\rangle$$

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$$\mathcal{H}_2 = \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1 \xrightarrow{\text{operate}} |\alpha\rangle$$

beam-splitter Hmlt

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(we will answer soon)

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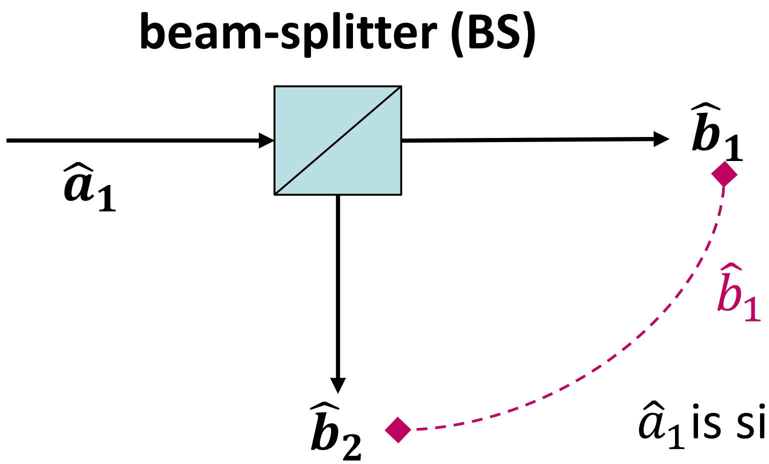
relation: single-mode noncls. & two-mode entangle.



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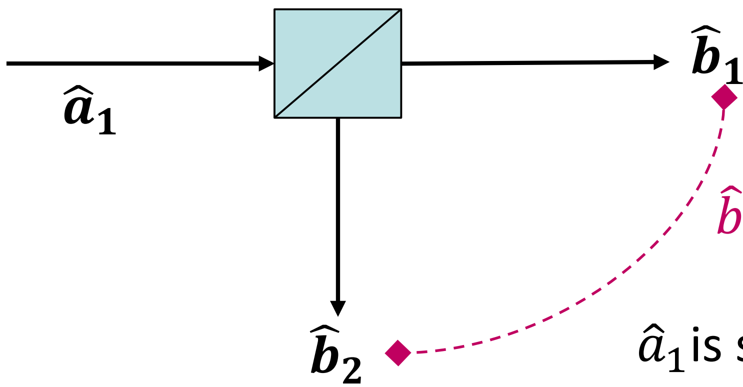

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use $\hat{b}_1$ & $\hat{b}_2$ entanglement criteria → ***obtain*** → single-mode nonclassicality criteria for $\hat{a}_1$

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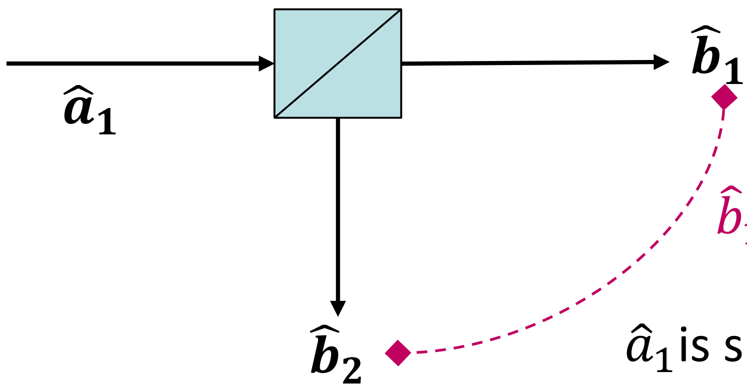

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quadrature squeezing criterion

DGCZ criterion

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[10] Mark Hillery and M Suhail Zubairy, Phys. Rev. A **74**, 032333 (2006).
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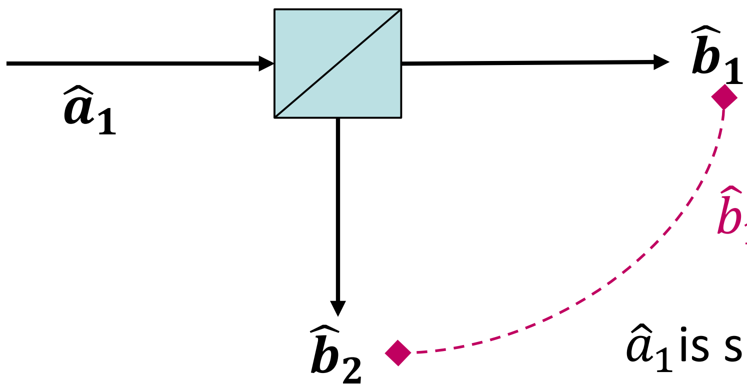

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H&Z criterion

number squeezing criterion

$\langle \hat{a}_1^\dagger \hat{a}_1 \hat{a}_2^\dagger \hat{a}_2 \rangle < |\langle \hat{a}_1^\dagger \hat{a}_2 \rangle|^2$ —— *implies* ——→ $\langle (\Delta \hat{n})^2 \rangle < \langle \hat{n} \rangle$

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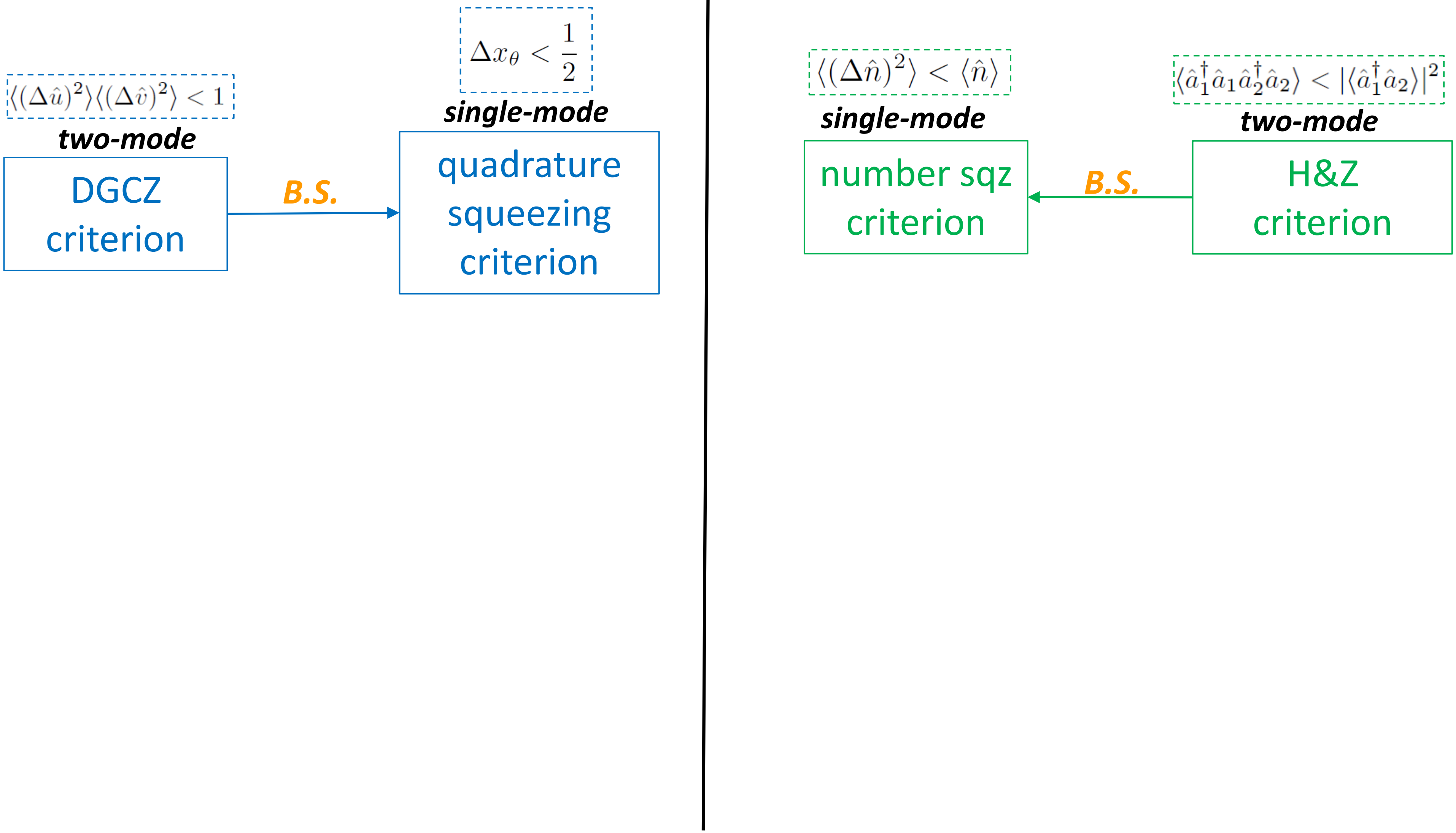

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$N \to \infty$ **[13,14]**

coherent states of light $|\boldsymbol{\alpha}\rangle$

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if $r \neq 1 \Rightarrow |\psi_N\rangle$ **inseparable**

if $r \neq 1 \Rightarrow |\psi\rangle$ **single-mode nonclassical**

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easier with operators

use Holstein-Primakoff transformation

$|e\rangle \dashrightarrow \hat{c}_e$

$|g\rangle \dashrightarrow \hat{c}_g$

$$S_+ = \hat{c}_e^\dagger\,\hat{c}_g$$

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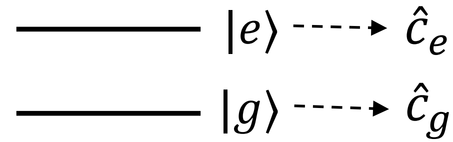

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$$S_+ = \hat{a}^\dagger \sqrt{N - \hat{a}^\dagger \hat{a}}$$

$$S_- = \sqrt{N - \hat{a}^\dagger \hat{a}} \; \hat{a}$$

$$S_z = \hat{a}^\dagger \hat{a} - N/2$$

representable with a single operator $\hat{a}$

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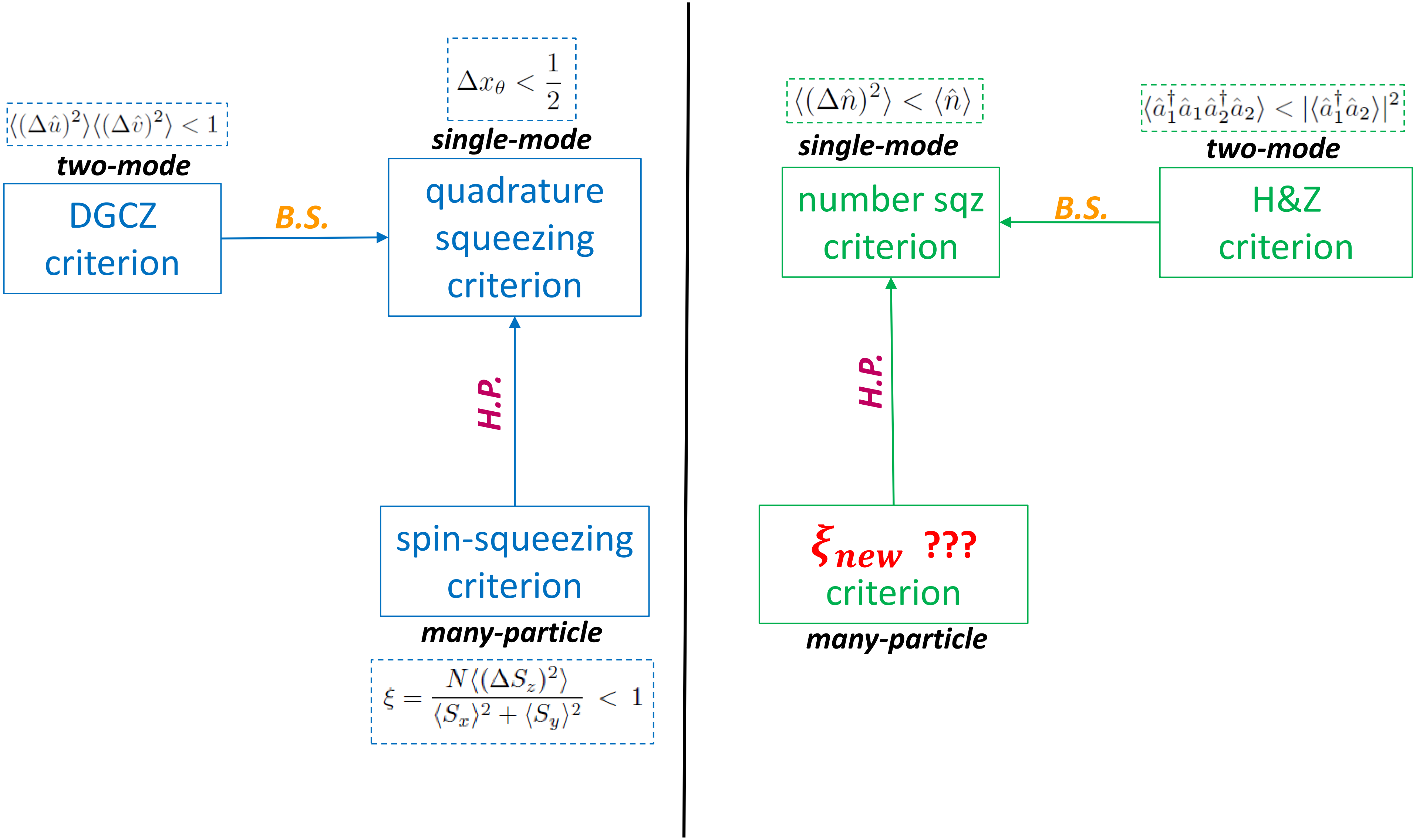
the big picture: (i) & (ii) & (iii) together



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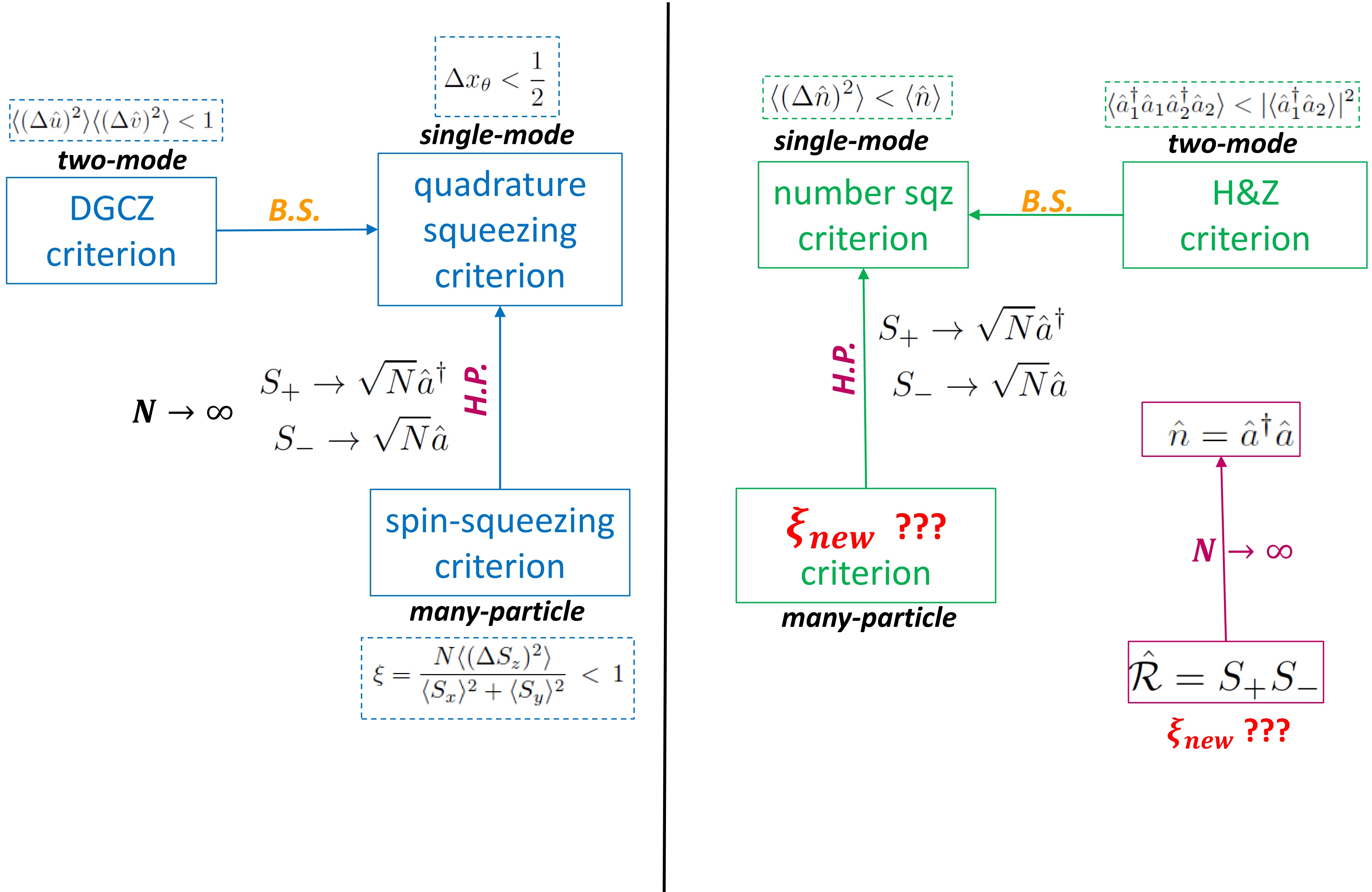

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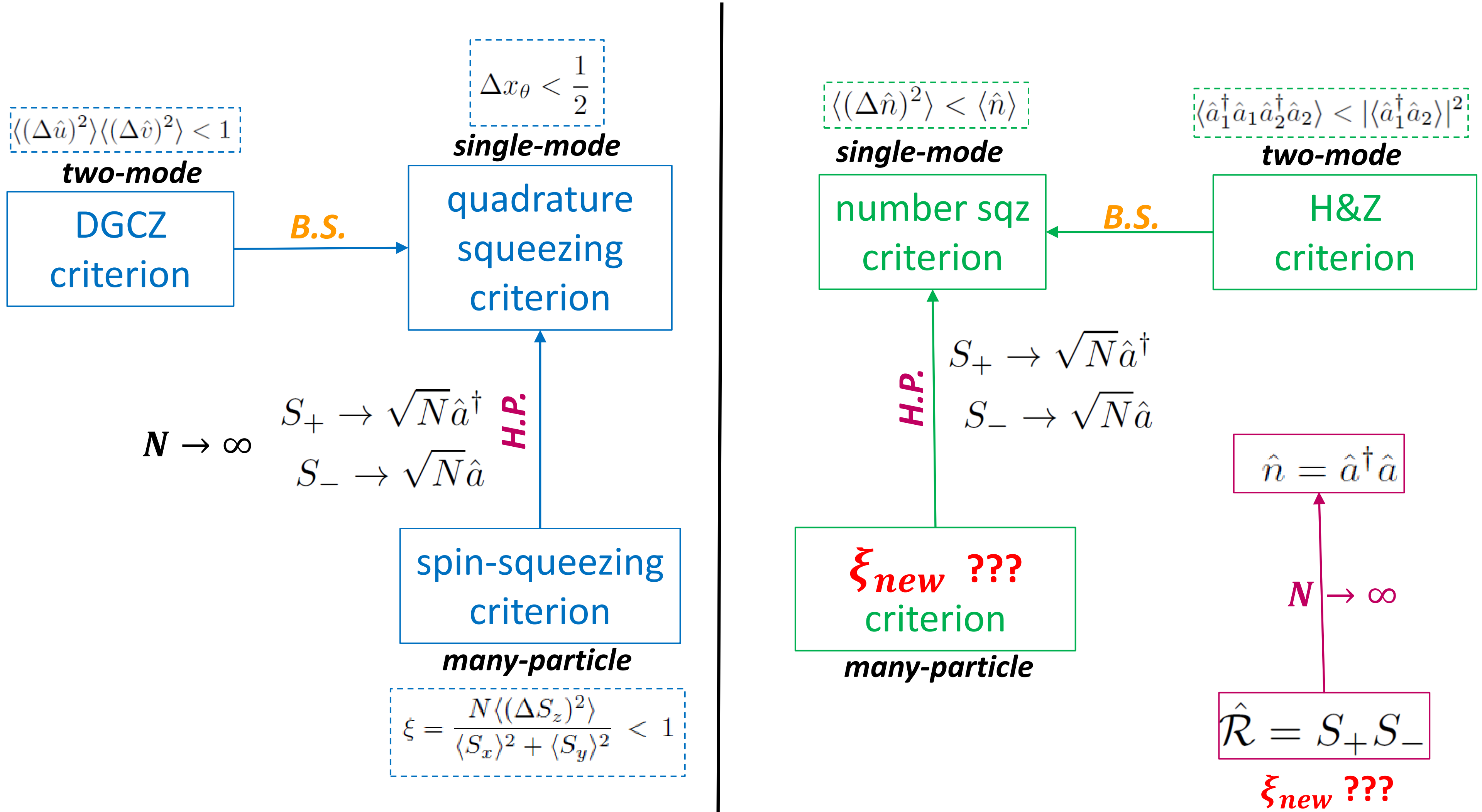


derived by calculating $\langle(\Delta\hat{S}_x)^2\rangle$

try calculating $\langle(\Delta\hat{R})^2\rangle = \langle(\Delta\hat{S}_+\hat{S}_-)^2\rangle$

new many-particle inseparability criterion: ξ_{new}

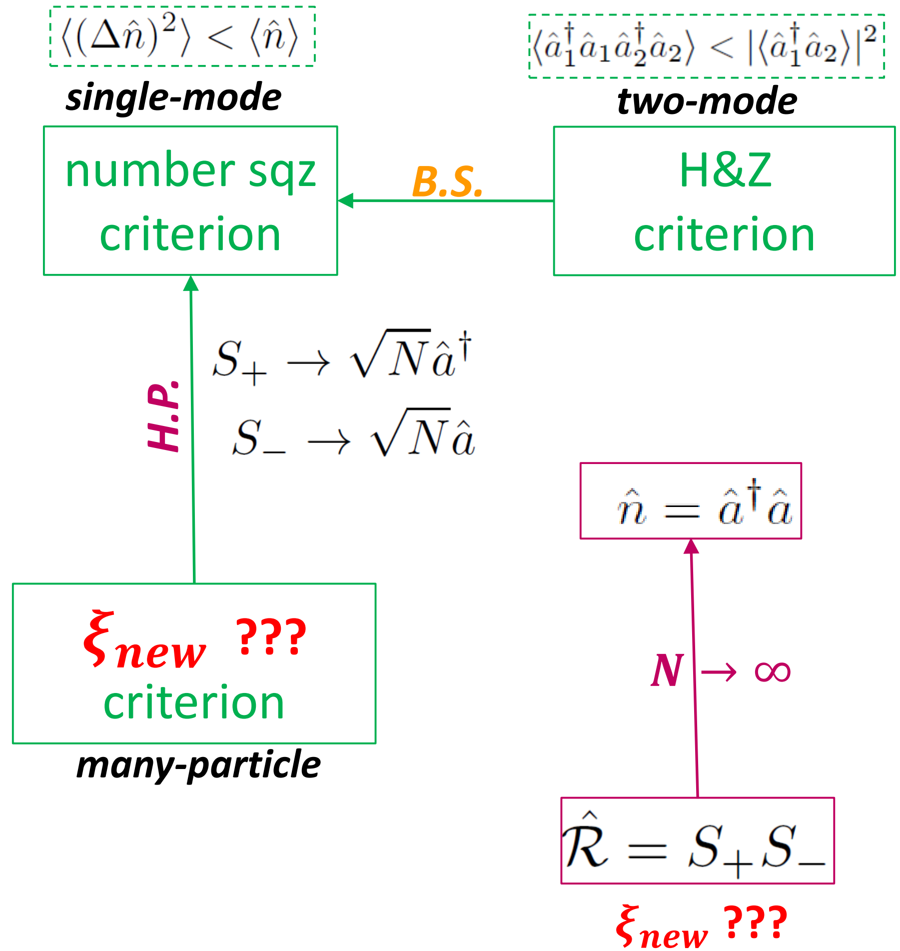
if separable

$$\hat{\rho} = \sum_k P_k \, \rho_1^{(k)} \otimes \rho_2^{(k)} \otimes \ldots \otimes \rho_N^{(k)}$$

$$\langle (\Delta \mathcal{R})^2 \rangle_\rho \geq \mu_N$$

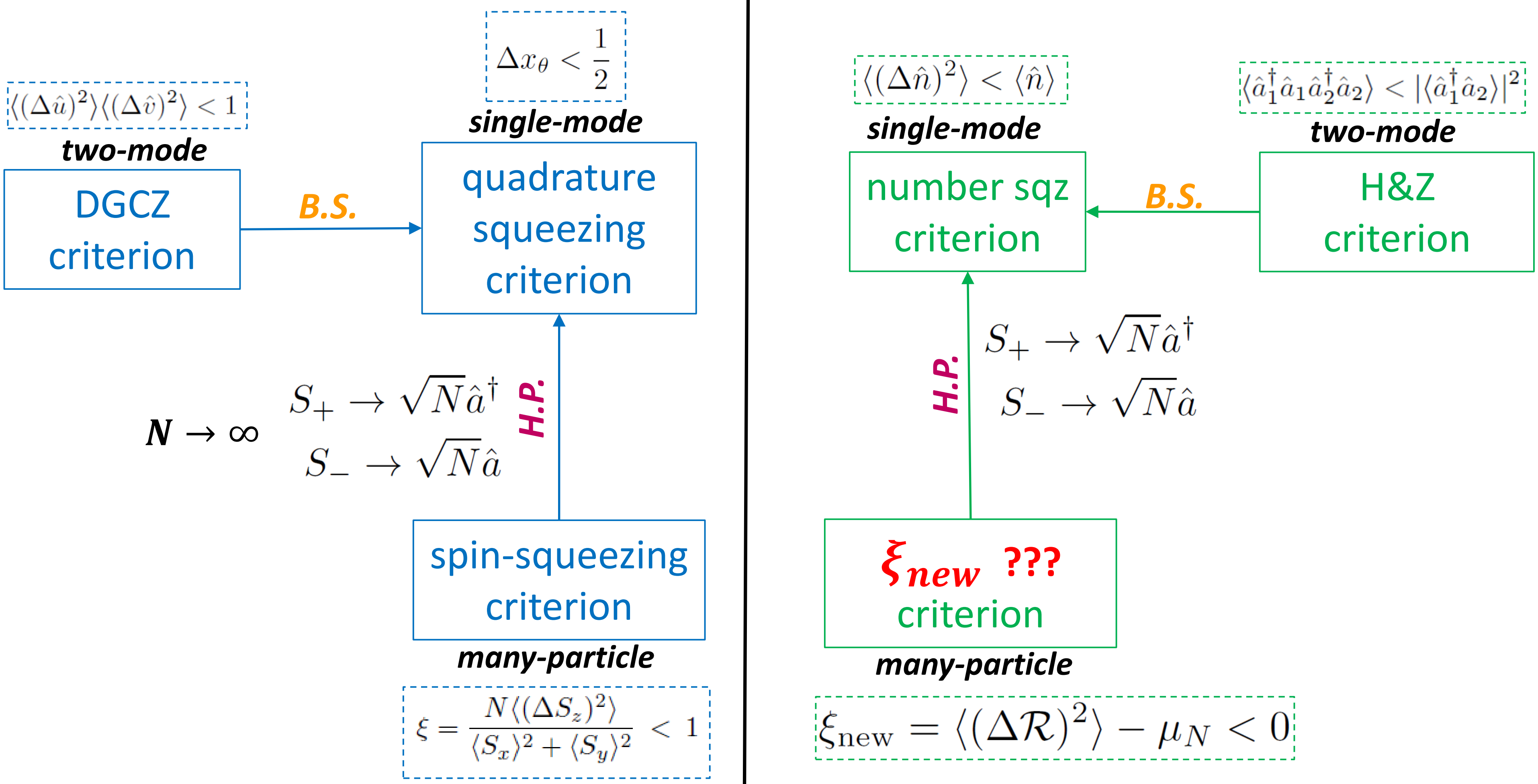
$$\xi_{\mathrm{new}} = \langle (\Delta \mathcal{R})^2 \rangle - \mu_N \geq 0$$

$\xi_{\mathrm{new}} < 0$ ⇒ **many-particle inseparable**



try calculating $\langle (\Delta \hat{R})^2 \rangle = \langle (\Delta \hat{S}_+ \hat{S}_-)^2 \rangle$

new many-particle inseparability criterion: ξ_{new}



derived by calculating $\langle(\Delta\hat{S}_x)^2\rangle$

try calculating $\langle(\Delta\hat{R})^2\rangle = \langle(\Delta\hat{S}_+\hat{S}_-)^2\rangle$

new many-particle inseparability criterion: ξ_{new}

$$\xi_{\rm new} = \langle (\Delta\mathcal{R})^2 \rangle - \mu_N < 0$$

$$\mu_N = \frac{\langle S_x \rangle_k^2 + \langle S_y \rangle_k^2}{N} + \frac{N^2}{4} - \langle S_z^2 \rangle_k + N\Big(\langle S_+ S_- \rangle_k - \langle S_z \rangle_k - \frac{N}{2} \Big)$$
$$-4[b^{1/2}(f_a^{1/2} + f_b^{1/2}) + f_a + f_b + 2 f_a^{1/2} f_b^{1/2}] - \frac{2}{N(N-1)}\Big(\langle S_x^2 \rangle + \langle S_y^2 \rangle - \frac{N}{2} \Big)^2$$

$$b = N/2 + \langle S_z \rangle_k - \left(\langle S_x \rangle_k^2 + \langle S_y \rangle_k^2 \right)/N \ ,$$

$$f_a = \frac{(N-1)^2}{4}\left(\frac{N}{4} - \frac{\langle S_y \rangle_k^2 + \langle S_z \rangle_k^2}{N} \right) - \frac{1}{4N}\Big(\langle S_x S_y + S_y S_x \rangle_k^2 + \langle S_x S_z + S_z S_x \rangle_k^2 \Big)$$

$$f_b = \frac{(N-1)^2}{4}\left(\frac{N}{4} - \frac{\langle S_x \rangle_k^2 + \langle S_z \rangle_k^2}{N} \right) - \frac{1}{4N}\Big(\langle S_x S_y + S_y S_x \rangle_k^2 + \langle S_y S_z + S_z S_y \rangle_k^2 \Big)$$

Outline

- Relation among
    i. single-mode nonclassicality
    ii. two-mode entanglement
    iii. many-particle entanglement

- new N-particle entanglement criterion
    i. test for Dicke states
    ii. test for the ground state Dicke Hamilonian (superradiance)
    iii. test for single-photon superradiance (exact, time depndt)
    iv. test for random superposition of Dicke states
    v. ground state of an interacting BEC

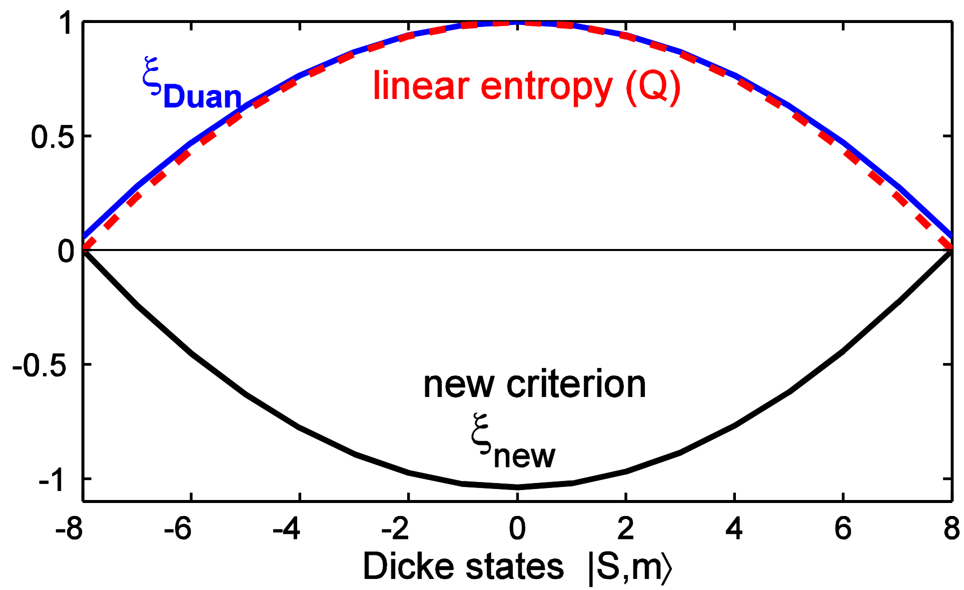
ξ_{new} Dicke states.

$$\xi_{\rm new} = \langle (\Delta \mathcal{R})^2 \rangle - \mu_N < 0$$

many-particle inseparable

$$Q > 0 \Rightarrow \text{entangled}$$

$$\xi_{\rm Duan} > 0 \Rightarrow \text{entangled}$$

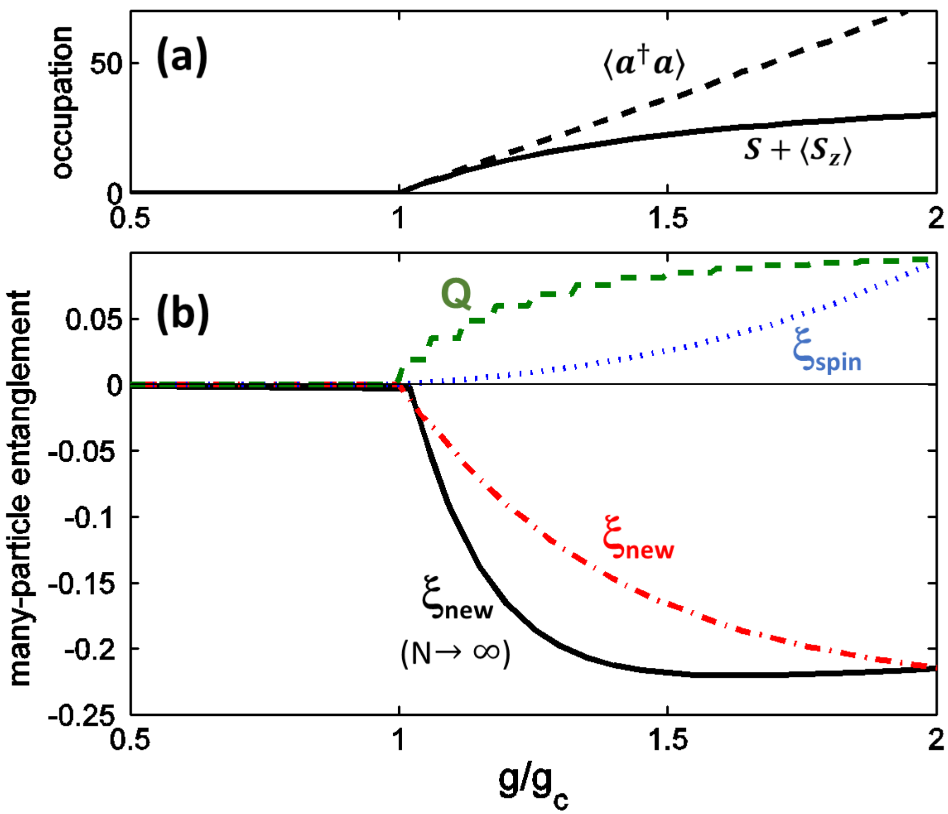


ξ_{new} **superradiance** (superpositions of Dicke states)

$$\xi_{\rm new} = \langle (\Delta\mathcal{R})^2\rangle - \mu_N < 0$$

ground state of the Dicke Hamiltonian (**superradiance**)

$$\hat{\mathcal{H}} = \hbar\omega_{eg}\hat{S}_z + \hbar\omega_a \hat{a}^\dagger\hat{a} + g/\sqrt{N}(\hat{S}_+ + \hat{S}_-)(\hat{a}^\dagger + \hat{a})$$



$Q > 0 \Rightarrow$ entangled

$\xi_{\rm spin} < 0 \Rightarrow$ entangled

$\xi_{new} < 0 \Rightarrow$ entangled

$\xi_{\rm Duan}$ also fails (not plotted)

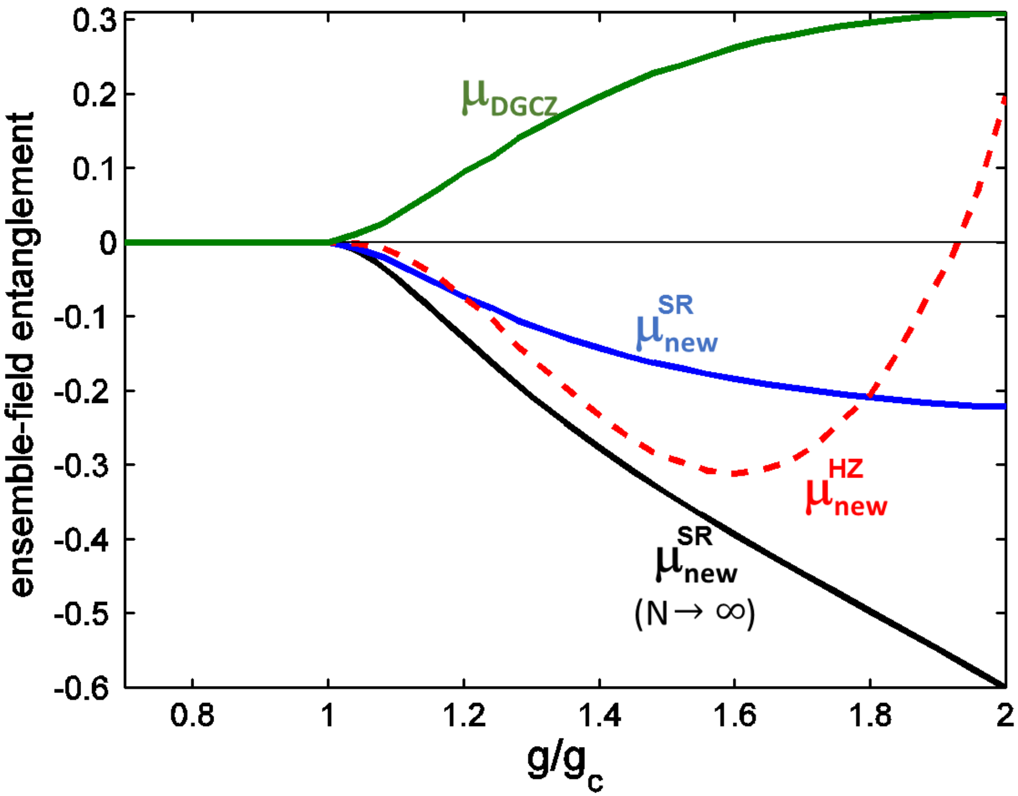
ξ_{new} **superradiance** (superpositions of Dicke states)

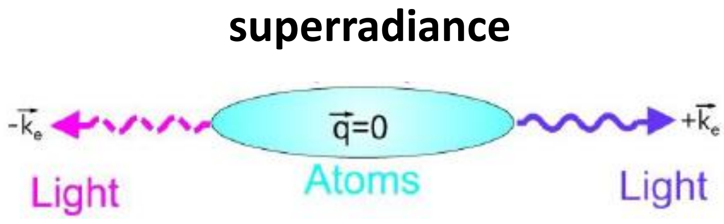
ensemble-field entanglement

$\mu_{new} < 0 \;\Rightarrow$ ensemble-field entanglement

ground state of the Dicke Hamiltonian (**superradiance**)

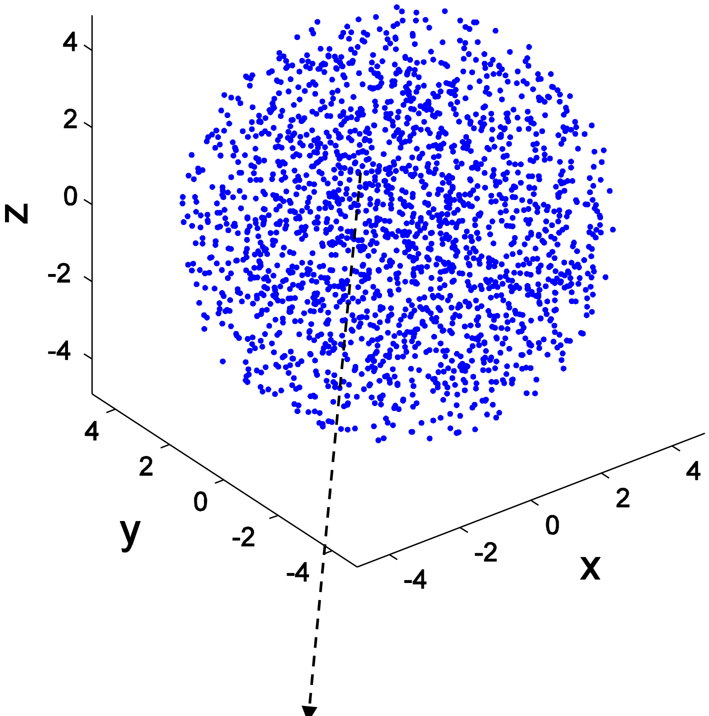
$$\hat{\mathcal{H}} = \hbar\omega_{eg}\hat{S}_z + \hbar\omega_a\hat{a}^\dagger\hat{a} + g/\sqrt{N}(\hat{S}_+ + \hat{S}_-)(\hat{a}^\dagger + \hat{a})$$





ξ_{new} single-photon superradiance: timed (superpositions of Dicke states)

single-photon superradiance



randomly placed 2000 atoms

exactly solvable

$$|\psi(0)\rangle = \sum_{j=1}^{N} e^{i\mathbf{k}_0\cdot\mathbf{r}_j}|g_1, g_2 \ldots e_j \ldots g_N\rangle$$

initially superposition of different atoms' excitations

$$|\psi(t)\rangle = \sum_{j=1}^{N} \beta_j(t)|g_1 g_2 \ldots e_j \ldots g_N\rangle|0\rangle + \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t)|g_1 g_2 \ldots g_N\rangle|1_{\mathbf{k}}\rangle$$

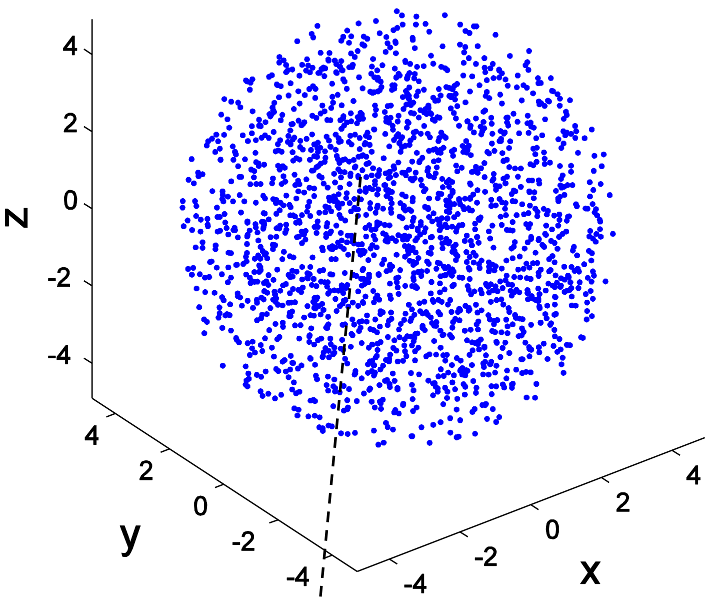
$$\beta_j(t) = \frac{1}{\sqrt{N}} e^{-\Gamma_N t} e^{i\mathbf{k}_0\cdot\mathbf{r}_j}$$

[16] Anatoly Svidzinsky and Jun-Tao Chang, "Cooperative spontaneous emission as a many-body eigenvalue problem," Phys. Rev. A **77**, 043833 (2008).

[17] Marlan O Scully, \Single photon subradiance: Quantum control of spontaneous emission and ultrafast readout," Physical Review Letters 115, 243602 (2015).

ξ_{new} single-photon superradiance: timed
(superpositions of Dicke states)

single-photon superradiance



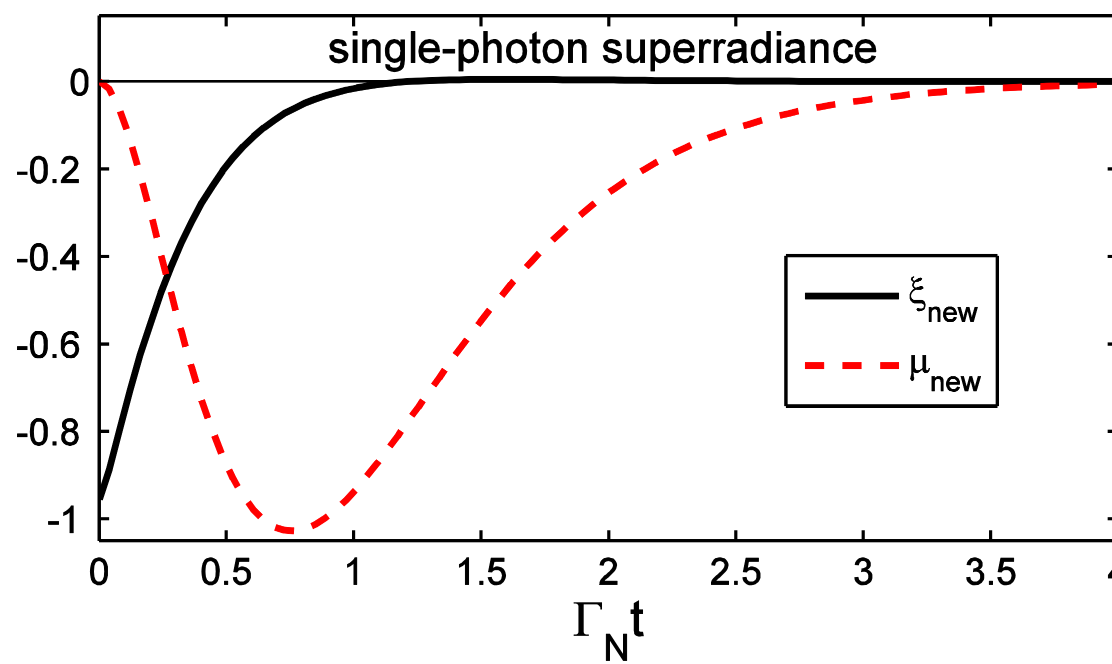
randomly placed 2000 atoms

exactly solvable

$$|\psi(0)\rangle = \sum_{j=1}^{N} e^{i\mathbf{k}_0\cdot\mathbf{r}_j}|g_1, g_2 \ldots e_j \ldots g_N\rangle$$

initially superposition of different atoms' excitations

$$|\psi(t)\rangle = \sum_{j=1}^{N} \beta_j(t)|g_1 g_2 \ldots e_j \ldots g_N\rangle|0\rangle + \sum_{\mathbf{k}} \gamma_{\mathbf{k}}(t)|g_1 g_2 \ldots g_N\rangle|1_{\mathbf{k}}\rangle$$



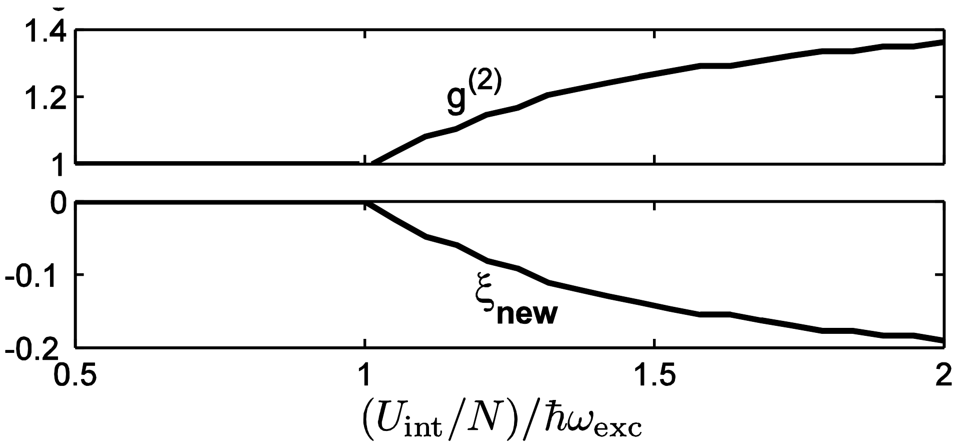
ξ_{new} (random superpositions of Dicke states)

2000 random superposition of Dicke states

witnessed in all when Q>0

ξ_{new} interacting BEC

$$\hat{\mathcal{H}} = \underbrace{\int d^3\mathbf{r}\hat{\psi}(\mathbf{r})^\dagger \hat{H}_0(\mathbf{r})\hat{\psi}(\mathbf{r})}_{\text{K.E. \& trap potential}} + g_s \underbrace{\int d^3\mathbf{r}\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})}_{\text{atom-atom collisions}}$$



$$g^{(2)} = \langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle$$

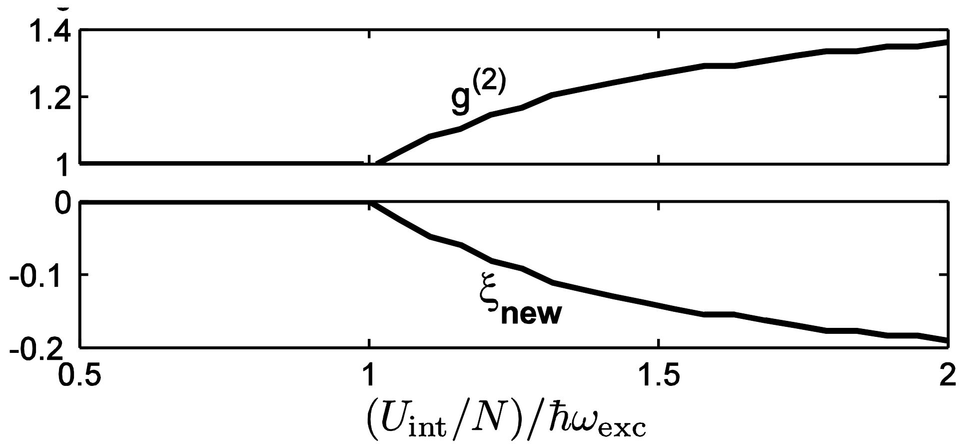
bunching of atoms

entanglement of atoms

ξ_{new} interacting BEC

$$\hat{\mathcal{H}} = \int d^3\mathbf{r}\hat{\psi}(\mathbf{r})^\dagger \hat{H}_0(\mathbf{r})\hat{\psi}(\mathbf{r}) + g_s \int d^3\mathbf{r}\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r})^\dagger\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})$$

K.E. & trap potential — atom-atom collisions



$$g^{(2)} = \langle\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}^\dagger(\mathbf{r})\hat{\psi}(\mathbf{r})\hat{\psi}(\mathbf{r})\rangle$$

bunching of atoms

entanglement of atoms

Ketterle [18] showed → **BEC responses *collectively* to an excitation if $U_{int} > E_{exc}$**

[18] J. Stenger, S. Inouye, Ananth P Chikkatur, DM Stamper-Kurn, DE Pritchard, and W. Ketterle, Bragg spectroscopy of a bose-einstein condensate," Phys. Rev. Lett. **82**, 4569 (1999).

*a question in place (**answer**)*

answer is in

$$S_+ \to \sqrt{N}\hat{a}^\dagger$$

$$S_- \to \sqrt{N}\hat{a}$$

becomes two-mode sqz

$$\mathcal{H}_1 = S_+ J_+ + S_- J_- \xrightarrow{\text{operate}} |\xi_{\text{ACS}}\rangle$$

⇨ generates **many-particle** entanglement

$$\mathcal{H}_2 = S_+ J_- + S_- J_+ \xrightarrow{\text{operate}} |\xi_{\text{ACS}}\rangle$$

becomes beam-splitter

⇨ cannot generate entanglement

$$\mathcal{H}_1 = \hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger \xrightarrow{\text{operate}} |\alpha\rangle$$

⇨ generates **two-mode** entanglement

$$\mathcal{H}_2 = \hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1 \xrightarrow{\text{operate}} |\alpha\rangle$$

beam-splitter Hmlt

⇨ cannot generate **two-mode** entanglement

[6] M. E. Tasgin and P. Meystre, “Spin sqz with coherent light via ent. swapping,” Phys. Rev. A **83**, 053848 (2011).

Acknowledgements

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