Quantum Thermodynamics with Rényi Entropy: Is it possible?

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Overview

- I. Rényi Entropy: Rudiments
- II. Quantum thermodynamics with Rényi Entropy: First Attempt
- III. Quantum thermodynamics with Rényi Entropy: Second Attempt
- IV. Concluding Remarks

Rényi Entropy: Rudiments

Rényi Entropy reads

$$S_\alpha(\rho) = \frac{1}{1-\alpha} \ln(Tr\rho^\alpha) \tag{1}$$

where $\alpha \in (0,1) \cup (1,\infty)$.

Note: von Neumann entropy as a limit!

$$S_{\alpha\to 1}(\rho) = -Tr(\rho \ln \rho) \tag{2}$$

Useful for

- Entanglement
- Dissipative systems
- Entropic uncertainty relations etc.

So,

- What about quantum thermodynamics?

Some more on Rényi Entropy

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Quantum thermodynamics with Rényi Entropy: First Attempt (I)

We maximize the Rényi Entropy subject to the constraints:

$$Tr\left(\rho\right) = 1 \tag{3}$$

and

$$Tr\left(\rho H\right) = U \tag{4}$$

Quantum thermodynamics with Rényi Entropy: First Attempt (II)

The thermal (UNFACTORIZED) equilibrium state then reads

$$\rho_\alpha = c\left[1 - (\alpha - 1)\beta H\right]^{\frac{1}{\alpha-1}} \tag{5}$$

So far, so good!
BUT, this is utterly useless!

Quantum thermodynamics with Rényi Entropy: First Attempt (III)

The thermal (FACTORIZED) equilibrium state then reads

$$\rho_\alpha = Z_\alpha^{-1} \left[1 - (\alpha - 1)\beta H\right]^{\frac{1}{\alpha-1}} \tag{6}$$

where

$$Z_\alpha = Tr\left[\left[1 - (\alpha - 1)\beta H\right]^{\frac{1}{\alpha-1}}\right].$$

Quantum thermodynamics with Rényi Entropy: First Attempt (IV)

However...

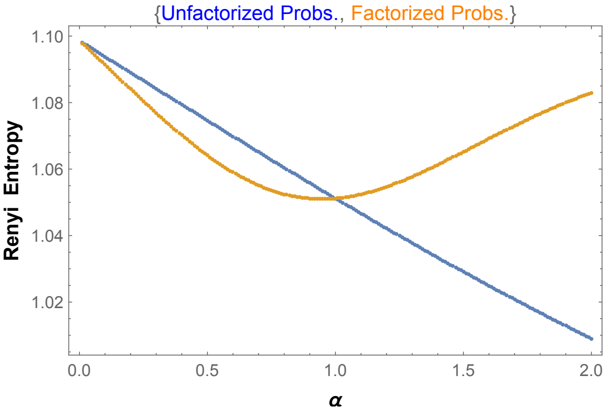

Figure: $H = |1\rangle\langle 1| + 3|2\rangle\langle 2| + 5|3\rangle\langle 3|$

Quantum thermodynamics with Rényi Entropy: Second Attempt (I)

The constraints now read:

$$Tr\,(\rho) = 1 \tag{7}$$

and

$$\frac{Tr\,(\rho^{\alpha} H)}{Tr\,(\rho^{\alpha})} = U_{\alpha} \tag{8}$$

Quantum thermodynamics with Rényi Entropy: Second Attempt (II)

The thermal equilibrium state now reads

$$\rho_\alpha = Z_\alpha^{-1} \left[1 - (1-\alpha)\beta \left(H - U_\alpha\right)\right]^{\frac{1}{1-\alpha}} \tag{9}$$

where

$$Z_\alpha = Tr\left[\left[1 - (1-\alpha)\beta \left(H - U_\alpha\right)\right]^{\frac{1}{1-\alpha}}\right].$$

Quantum thermodynamics with Rényi Entropy: Second Attempt (III)

However,

$$Tr\rho^{\alpha} = Z_{\alpha}^{1-\alpha} \tag{10}$$

so that

$$S_{\alpha}(\rho) = \frac{1}{1-\alpha}\ln\left(Tr\rho^{\alpha}\right) = \ln Z_{\alpha} \tag{11}$$

Quantum thermodynamics with Rényi Entropy: Second Attempt (IV)

So that we have

$$F_\alpha = -TS_\alpha \quad (12)$$

which can be correct ONLY when

$$U_\alpha = 0. \quad (13)$$

- The Rényi Entropy can be used for entanglement, dissipative systems, entropic uncertainty relations etc.
- The Rényi Entropy cannot be used in quantum thermodynamics with $Tr\,(\rho H) = U$, because the partition function cannot be identified!
- The Rényi Entropy cannot be used in quantum thermodynamics with $\frac{Tr(\rho^\alpha H)}{Tr(\rho^\alpha)} = U_\alpha$, because there can be no consistent thermodynamics e.g. $F_\alpha = -TS_\alpha$!

References

Thomas Oikonomou and G. Baris Bagci (2017)
Misusing the entropy maximization in the jungle of generalized entropies
Physics Letters A 381(4), 207 – 211.

Thank You!