Optimal Entanglement for Many-Body Systems via Quantum Correlations

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METU

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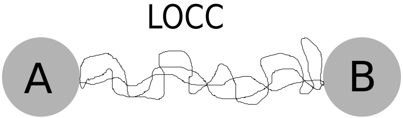
What's different about manybody entanglement?

Figure: A standart QIP picture



Plenio and Virmani, quant-ph/0504163

Figure: A standart QIP picture

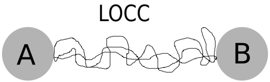
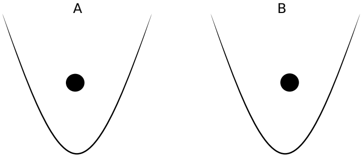


Figure: Two potential wells

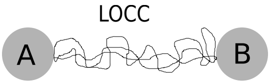


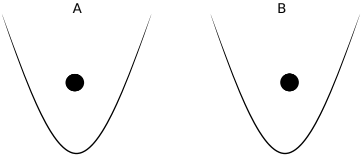
- Lack of natural partitioning

V. Vedral, CEJP 2 (2003) 289-306

- Lack of natural partitioning
    - Ambiguity in tensor product structure (TPS)

Zanardi et. al. PRL, 92, 060402
Caban et. al. J.Phys.A, 38, L79-L86
Ghirardi and Marinatto, PRA, 70, 012109





$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

What's the TPS of Schrödinger's cat?

- Lack of natural partitioning
  - Ambiguity in tensor product structure (TPS)
- Particle identity

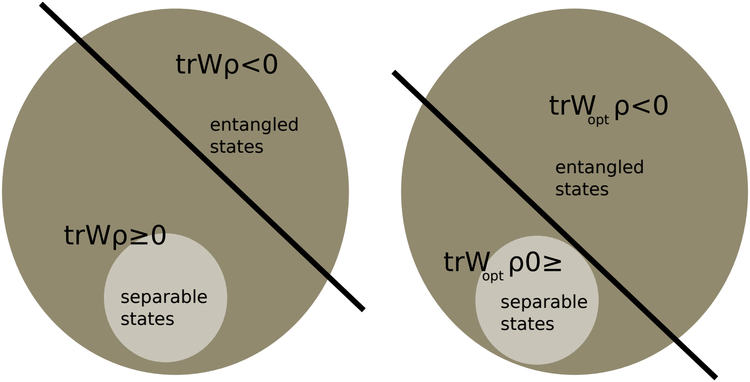
Identical Particles

$$\psi_A(x)\psi_B(y) \pm \psi_A(y)\psi_B(x)$$

- Labeling is not natural, particle labels are pseudo
- Slater determinants or permanents
- Operationally inaccessible
- Can be registered on different parties

Then, how to quantify entanglement of identical particles? Schliemann et. al. PRA, 64, 022303; Paskauskas and You, PRA 64, 042310; Wisemann and Vaccaro, PRL, 91, 097902; V. Vedral, CEJP 2 (2003) 289-306

Entanglement Witnessing



Entanglement Witnessing

$$(i) \left\langle \hat{W} \right\rangle = tr \left(\rho_{sep} \hat{W} \right) \geq 0 \qquad \forall \, \rho_{sep} \in \mathcal{S}$$

$$(ii) \left\langle \hat{W} \right\rangle = tr \left(\rho \hat{W} \right) < 0 \qquad \exists \, \rho \notin \mathcal{S}$$

then $\hat{W}$ is a witness operator

Barbara Terhal, Theoretical Computer Science, 287, 313-335
Lewenstein et. al. PRA, 63, 044304

Optimal Witness

$\hat{C}$ is a positive Hermitian operator, then $\hat{W} = \mathbb{1}\lambda - \hat{C}$ is optimal iff $\lambda = \sup tr \rho_{sep} \hat{C}$

J. Sperling, W. Vogel PRA 79, 022318

Fermionic Separability

$$|\Psi\rangle = c_1^\dagger c_2^\dagger \cdots c_N^\dagger |0\rangle$$

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Use observables of the form $Q^\dagger Q$ where

$$Q = \sum_{i=1}^{n} c_i c_{i+n} = c_1 c_{n+1} + \cdots + c_n c_{2n}$$

Fermionic Separability

$$\Lambda_{n,N}^{\mathrm{sep}} = \sup_{\rho_{\mathrm{sep}}} \mathrm{tr}\, \rho_{\mathrm{sep}} Q^\dagger Q = \sup_{|\Psi\rangle \ \mathrm{unent.}} \langle \Psi | Q^\dagger Q | \Psi \rangle$$

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The separable state limit of $\langle Q^\dagger Q \rangle$ for N-fermion states

$$\Lambda_{n,N}^{\mathrm{sep}} = \begin{cases} \lfloor \frac{N}{2} \rfloor & \mathrm{for}\ N < 2n \\ n & \mathrm{for}\ N \geq 2n \end{cases}$$

$\lfloor x \rfloor$ denotes the integer-part function

Fermionic Separability

We have to show that there are states $\langle Q^\dagger Q \rangle > \Lambda^{\mathrm{sep}}_{n,N}$

$$|\Psi_m\rangle = (Q^\dagger)^m c^\dagger_{k_1} c^\dagger_{k_2} \cdots c^\dagger_{k_{N-2m}} |0\rangle$$

where $k_1, \ldots, k_{N-2m}$ is in the list $1, \ldots, 2n$, then

$$Q^\dagger Q |\Psi_m\rangle = m(n + 1 - m - p_{N_Q}) |\Psi_m\rangle \qquad (1)$$

where $p_{N_Q} = 0$ (1) if N_Q is even (odd), finally

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$$\lambda_{n,N} = \max_{0 \le m \le n, N/2} m(n+1-m) \tag{2}$$

$$= \begin{cases} \lfloor \frac{N}{2} \rfloor (n+1-\lfloor \frac{N}{2} \rfloor) & \text{for } N < 2\lfloor \frac{n+1}{2} \rfloor \,, \\ \lfloor \frac{n+1}{2} \rfloor \lceil \frac{n+1}{2} \rceil & \text{for } N \ge 2\lfloor \frac{n+1}{2} \rfloor \,, \end{cases} \tag{3}$$

$\lceil x \rceil$ shows the smallest integer greater than or equal to x

For the cases $n > 2$ and $N \geq 2$ the strict inequality $\lambda_{n,N} > \Lambda^{\mathrm{sep}}_{n,N}$ holds, that's good

For $n = 2$ and $N \geq 4$, we have $\lambda_{2,N} = \Lambda^{\mathrm{sep}}_{2,N} = 2$, this is bad