Using Topological Stability to Process Quantum Information

Canberk Şanlı

Bogazici University

canberk.sanli@boun.edu.tr

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Motivation

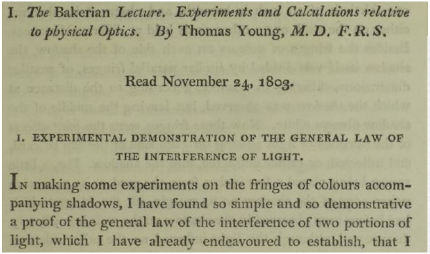
More Honest Title :
Review of Josephson Junctions from 60's to our date

- What is a macroscopic quantum phenomena ?
- Role of topological invariants in the macroscopic quantization
- How is it used in quantum information processing ?

Overview

Double Slit Experiment

- Double Slit Experiment with light waves

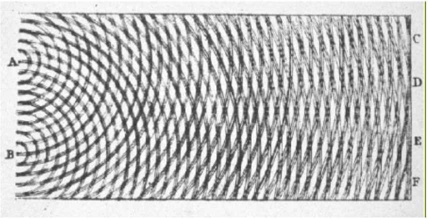


I. *The* Bakerian *Lecture. Experiments and Calculations relative to physical Optics.* *By* Thomas Young, *M. D. F. R. S.*

Read November 24, 1803.

I. EXPERIMENTAL DEMONSTRATION OF THE GENERAL LAW OF THE INTERFERENCE OF LIGHT.

In making some experiments on the fringes of colours accompanying shadows, I have found so simple and so demonstrative a proof of the general law of the interference of two portions of light, which I have already endeavoured to establish, that I



Double Slit Experiment with electrons

- Double Slit Experiment with Electrons

Aus dem Institut für Angewandte Physik der Universität Tübingen

Elektroneninterferenzen an mehreren künstlich hergestellten Feinspalten

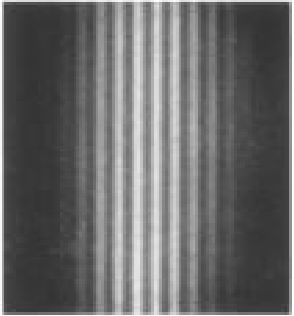
Von

Claus Jönsson

Mit 14 Figuren im Text

(Eingegangen am 17. Oktober 1960)

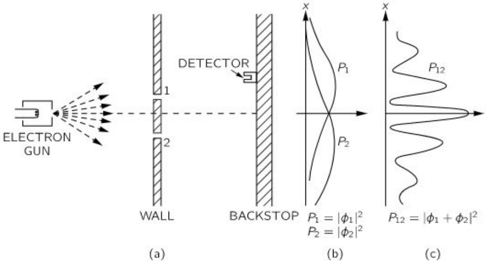
A glass plate covered with an evaporated silver film of about 200 Å thickness is irradiated by a line-shaped electron-probe in a vacuum of 10^{-4} Torr. A hydrocarbon polymerisation film of very low electrical conductivity is formed at places subjected to high electron current density. An electrolytically deposited copper film leaves these places free from copper. When the copper film is stripped a grating with slits free of any material is obtained. 50 μ long and 0·3 μ wide slits with a grating constant of 1 μ are obtained. The maximum number of slits is five. The electron diffraction pattern obtained using these slits in an arrangement analogous to Young's light optical interference experiment in the Fraunhofer plane and Fresnel region shows an effect corresponding to the well-known interference phenomena in light optics.



Elektronenbeugungsaufnahme an zwei Spalten (Fraunhofer-Ebene)

Double Slit Experiment with electrons

- NOT a ‘do at home’ experiment



- coherence of particles
- apparatus of impossibly small scale

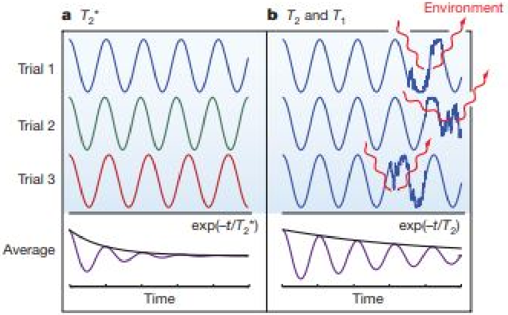
No observer in Schrödinger equation

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

- Standard QM only applies to closed systems
- why macroscopic superpositions cannot be observed
  $\Rightarrow$ Decay so quickly
  $\Rightarrow$ main problem of building Quantum Computers

Hardware of a quantum computer

- still ongoing research
- various candidates : photon polarizations, trapped ions, nuclear spin in molecules, quantum dots and dopants in solids, quantum circuits
- how to compare the alternatives : Decoherence & Dephasing



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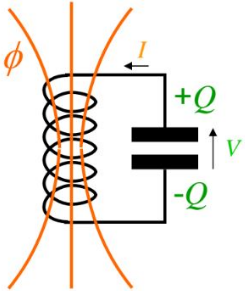
[1] [Ladd et al.(2010)Ladd, Jelezko, Laflamme, Nakamura, Monroe, and OBrien]

Quantum Circuits

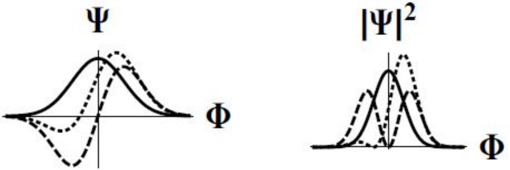
- First attempt : quantize classical LC oscillator



$$H = \frac{Q^2}{2C} + \frac{\Phi^2}{2L}, \quad \Rightarrow \quad [\hat{Q}, \hat{\Phi}] = i\hbar$$

$$H = \frac{\hbar\Omega}{2}[a^\dagger a + aa^\dagger] \; ; \; \Omega = 1/\sqrt{LC}$$

- well defined quantum states



- why this circuit cannot be used as a qbit :
  Resistance ⇒ Heat dissipation ⇒ any quantum effect disappears
- Solution : use superconductors → zero resistivity

Superconductivity

at $T < T_c$, most metals :

- Resistanceless to electric flow
- Expulsion of magnetic field
- Trapped flux quantization
  $\Rightarrow$ A Thermodynamic State
- Number of available states $\downarrow$ as $T \to 0$
  $\Rightarrow$ Macroscopic quantum phenomena

- Local conservation of probability

$$\nabla.\vec{J} = -\frac{\partial P}{\partial t}$$

$$\vec{J} = \frac{1}{2}\left\{\Psi^*\left[\frac{\hat{p}-q\vec{A}}{m}\right]\Psi + \Psi\left[\frac{\hat{p}-q\vec{A}}{m}\right]^*\Psi^*\right\}, \quad P = \Psi\Psi^*$$

- what happens when there are large number of particles in the same state
- $|\Psi > \sim \Big|\text{classical, macroscopic situation}\Big\rangle$

$\Psi_{superconductor}$

$$\vec{J} = \frac{1}{2}\left\{\Psi^*\left[\frac{\hat{p} - q\vec{A}}{m}\right]\Psi + \Psi\left[\frac{\hat{p} - q\vec{A}}{m}\right]^*\Psi^*\right\} , \;\; P = \Psi\Psi^*$$

- large number of particles in the same state
- $|\Psi> = \sqrt{\rho(r)}e^{i\theta(r)}$
- $q_e\Psi\Psi^*$: charge density
- $\vec{J}$: electric current density

$$\vec{J} = \frac{\hbar}{m}(\nabla\theta - \frac{q}{\hbar}\vec{A})\rho$$

Hallmarks of Superconductivity

- Bose condensate of pairs
  $\Rightarrow$ Resistanceless flow & Expulsion of magnetic field
- $|\Psi >= \sqrt{\rho(r)}e^{i\theta(r)}$
- $q_e \Psi\Psi^*$: charge density
- $\vec{J}$: electric current density

$$\begin{aligned} \nabla.\vec{J} &= \nabla.\frac{\hbar}{m}(\nabla\theta - \frac{q}{\hbar}\vec{A})\rho \\ \vec{J} &= -\rho\frac{q}{\hbar}\vec{A} \\ A &= ce^{-\lambda x} \end{aligned}$$

Hallmarks of Superconductivity

- Meissner effect

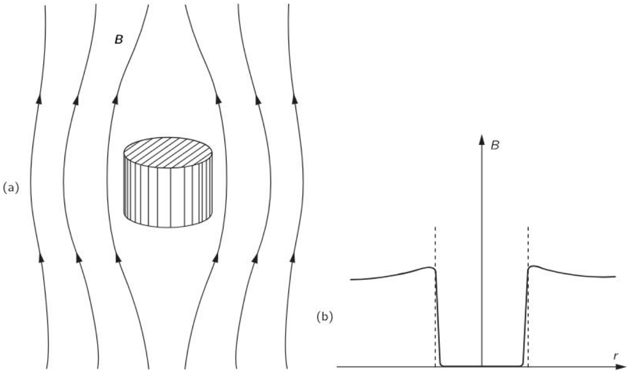


Fig. 21–3. (a) A superconducting cylinder in a magnetic field; (b) the magnetic field B as a function of r. [2]

[2] [Feynman and Leighton(1964)]

- flux quantization in a loop superconductor

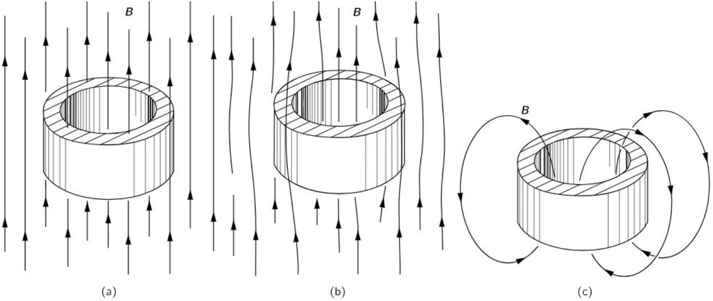


Fig. 21–4. A ring in a magnetic field: (a) in the normal state; (b) in the superconducting state; (c) after the external field is removed. [3]

[3] [Feynman and Leighton(1964)]

Hallmarks of Superconductivity

- flux quantization in a loop superconductor

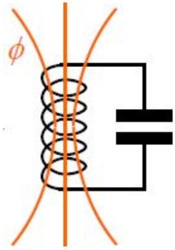
$$\vec{J} = \frac{\hbar}{m}(\nabla\theta - \frac{q}{\hbar}\vec{A})\rho$$

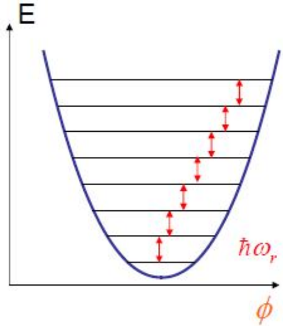
$$\int_{\Gamma} \vec{J}.\vec{dl} = 0 \;\Rightarrow\; \Phi = \frac{2\pi\hbar}{q} n = \Phi_0 n \;;\; n = 1, 2, ..$$

- flux is trapped in quantized amounts !

Quantum LC Circuit

- Still, not enough :

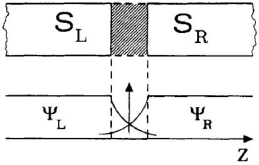




- All transitions are degenerate ! Not suitable for qubit implementation.
- Solution : Need a non-linear circuit element → Josephson Junction.

Josephson Junction

- weak coupling of two superconductors at a distance of 1 nm.
- $\Psi_{L,R}$: amplitude to find an electron on side $L, R = \sqrt{\rho}e^{i\theta_{L,R}}$
- $i\hbar\frac{\partial\Psi_L}{\partial t} = \frac{qV}{2}\Psi_L + K\Psi_R$, $i\hbar\frac{\partial\Psi_R}{\partial t} = \frac{qV}{2}\Psi_R + K\Psi_L$.



- $J = J_0 \sin\delta$; $\delta = \delta_0 + \frac{q}{\hbar}\int V(t)dt$, or $\delta = \delta_0 + \frac{q}{\hbar}\int \vec{A}.\vec{dl}$

JOSEPHSON CURRENTS IN SUPERCONDUCTING TUNNELING: THE EFFECT OF MICROWAVES AND OTHER OBSERVATIONS*

Sidney Shapiro
Arthur D. Little, Inc., Cambridge, Massachusetts
(Received 13 June 1963)

In the course of experiments on the effect of microwave fields on superconducting tunneling, we have had occasion over the past few months to fabricate many tunneling crossings of low resistance (5-20 Ω with a crossing area of 1.5 $\times 10^{-4}$ cm^2). Every one of these samples has exhibited the zero-voltage currents predicted by Josephson[1] and attributed, in effect, to the tunneling of Cooper pairs. The observation of these currents has already been reported by Anderson and Rowell.[2] Our experiments have brought to

No attempt was made to shield the earth's magnetic field. Most data were taken at about 0.9°K.

The following observations were noted in the course of experiments with a large number of tunneling crossings:

1. Using an ac display, Fig. 1 shows for a typical sample the zero-voltage current predicted by Josephson and previously observed with a dc technique by Anderson and Rowell. During each half-cycle of the sweep, current

Josephson Junction

- $J = J_0 \sin\delta$; $\delta = \delta_0 + \frac{q}{\hbar}\int V(t)dt$, or $\delta = \delta_0 + \frac{q}{\hbar}\int \vec{A}.\vec{dl}$
- non-zero net current at V=0, zero net current at $V \neq 0$
- except at certain frequencies→ Shapiro steps.

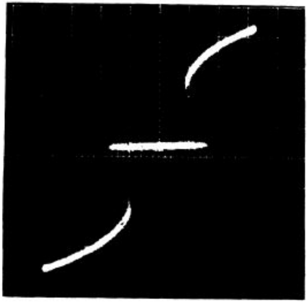


FIG. 1. *I*-*V* characteristic near origin showing zero-voltage Josephson current and negative resistance switching trace. Vertical scale 58.8 μV/cm, horizontal scale 130 nA/cm.

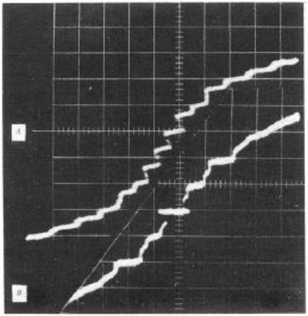
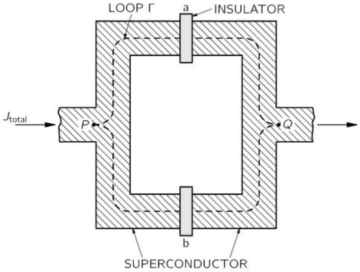


FIG. 3. Microwave power at 9300 Mc/sec (*A*) and 24850 Mc/sec (*B*) produces many zero-slope regions spaced at $h\nu/2e$ or $h\nu/e$. For *A*, $h\nu/e = 38.5$ μV, and for *B*, 103 μV. For *A*, vertical scale is 58.8 μV/cm, horizontal scale is 67 nA/cm; for *B*, vertical scale is 50 μV/cm, horizontal scale is 50 μA/cm.

- Calculate $\Delta\text{Phase}_{P \to Q}$ from two ways.



- $J_{max} = J_0 \left| \cos \frac{q_0 \Phi}{\hbar} \right|; \ \Phi^* = n\Phi_0$

Electrodynamics of JJ

- $J = J_0 \sin\delta$; $\delta = \delta_0 + \frac{q}{\hbar}\int \vec{A}.\vec{dl}$, $\nabla x \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
- Sine-Gordon Equation :

$$\frac{\partial^2 \delta}{\partial x^2} - \frac{\partial^2 \delta}{\partial t^2} = \sin\delta$$

- $\mathcal{L} = \frac{1}{2}(\delta_x^2 - \delta_t^2) - (1 - \cos\delta)$
- $E[\delta] = \int_{-\infty}^{\infty} dx \left[\frac{\dot{\delta}^2}{2} + \frac{\delta'^2}{2} + 1 - \cos\delta \right]$

- Seperate the boundary term (Bogomolny trick)

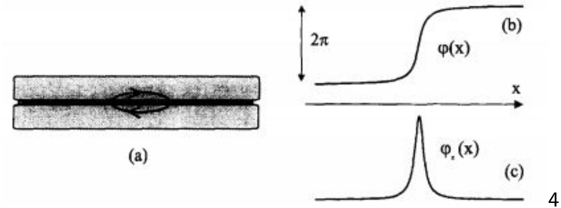
$$E[\delta] = \int_{-\infty}^{\infty} dx \left[\frac{\dot{\delta}^2}{2} + \frac{1}{2}\left(\delta' + \sqrt{2(1-\cos\delta)}\right)^2 - \sqrt{2(1-\cos\delta)\delta'} \right]$$

$$E[\delta] = \int_{-\infty}^{\infty} dx \left[\frac{\dot{\delta}^2}{2} + \frac{1}{2}\left(\delta' + \sqrt{2(1-\cos\delta)}\right)^2 \right] + [W(\delta(\infty)) - W(\delta(-\infty))]$$

- BPS equations : $\dot{\delta} = 0$, $\delta' = \sqrt{2(1-\cos\delta)}$
- Non-trivial solution : $\delta = 4\tan^{-1}[\exp(\frac{(x-x_0)-ut}{\sqrt{1-u^2}})]$
- Associated conserved charge : $N = \frac{1}{2\pi}\int_{-\infty}^{\infty} \delta' dx$

Solitons in JJ

- $\delta = 4\tan^{-1}[\exp(\frac{(x-x_0)-ut}{\sqrt{1-u^2}})]$, $N = \frac{1}{2\pi}\int_{-\infty}^{\infty}\delta' dx$, $\Phi = \Phi_0 N$

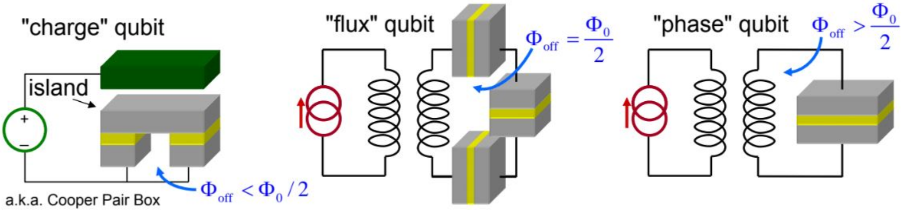


[4]

- **SLOGAN :** Soliton = Finite energy field configuration that cannot decay to vacua (lowest energy) configurations

[4] [Ustinov(1998)]

- Josephson energy : $U = \int_0^t IVdt = \frac{\Phi_0 I_0}{2\pi}(1 - \cos\delta) = E_J(1 - \cos\delta)$
- Combination of Capacitance, Conventional inductance, JJ, bias
- Quantum dynamics ✓ , Coherence ✓, Non-linearity ✓
- E_J/E_C determines :

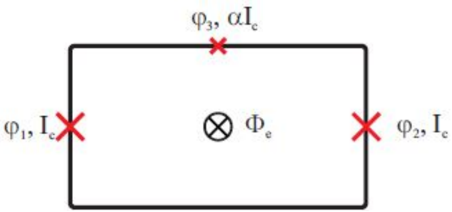


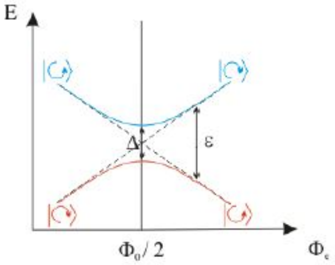
[5]

[5] [Devoret and Martinis(2005)]

Flux qubit

- Three Josephson Junctions
- tunneling process of a Josephson phase between two neighboring shallow potential wells : $\Phi \approx 0.5\Phi_0 \Rightarrow |\Psi>=|\circlearrowright> + |\circlearrowleft>$: Screen or enhance.





[6]

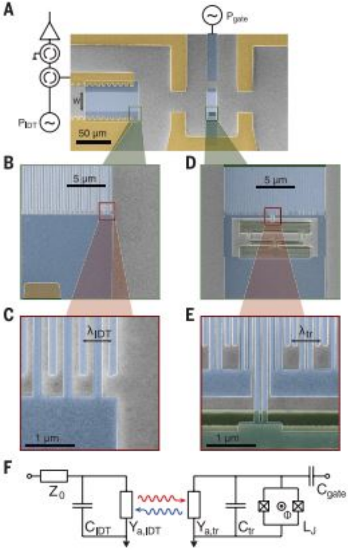
[6][Fedorov et al.(2014)Fedorov, Shcherbakova, Wolf, Beckmann, and Ustinov]

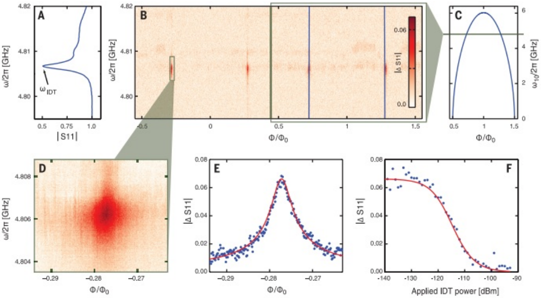
Science 346 (6206), 207-211.
DOI: 10.1126/science.1257219originally published online September 11, 2014

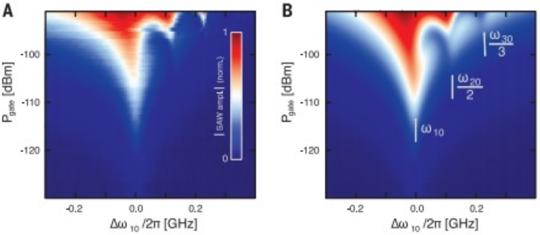
Propagating phonons coupled to an artificial atom

Martin V. Gustafsson,[1,2*] Thomas Aref,[1] Anton Frisk Kockum,[1] Maria K. Ekström,[1] Göran Johansson,[1] Per Delsing[1*]

Quantum information can be stored in micromechanical resonators, encoded as quanta of vibration known as phonons. The vibrational motion is then restricted to the stationary eigenmodes of the resonator, which thus serves as local storage for phonons. In contrast, we couple propagating phonons to an artificial atom in the quantum regime and reproduce findings from quantum optics, with sound taking over the role of light. Our results highlight the similarities between phonons and photons but also point to new opportunities arising from the characteristic features of quantum mechanical sound. The low propagation speed of phonons should enable new dynamic schemes for processing quantum information, and the short wavelength allows regimes of atomic physics to be explored that cannot be reached in photonic systems.




A
B
C
D
E
F
ω/2π [GHz]
|S11|
ω_IDT
Φ/Φ0
|Δ S11|
ω10/2π [GHz]
Applied IDT power [dBm]


A
B
P_gate [dBm]
Δω 10 /2π [GHz]
|SAW ampl.| (norm.)
ω 10
ω 20 / 2
ω 30 / 3

Topological JJ

- Topological superconductor : existence of Majorana fermions
- Topological JJ : made from topological superconductor
- more strong against dephasing, larger coherence time
- why ?
  1) ohmic dissipation suppresses quantum tunnelling : $\Delta N \Delta\delta \approx 2\pi$
  $\Rightarrow$ some ohmic dissipation must be allowed to have a coherent phase
  2) two different dissipation mechanism balancing each other

$$V(\delta) = \mu[1 - \cos\delta] + \lambda[1 - \cos\delta/2] \tag{1}$$

$\Rightarrow$ phase coherence is achieved at a smaller ohmic dissipation
$\Rightarrow$ coherent qubit

PRL **112,** 247001 (2014) PHYSICAL REVIEW LETTERS week ending 20 JUNE 2014

Dissipation in a Simple Model of a Topological Josephson Junction

Paul Matthews,[1,2] Pedro Ribeiro,[3,4,5] and Antonio M. García-García[6,5]
[1]*CIC nanoGUNE, Tolosa Hiribidea 76, 20018 Donostia-San Sebastian, Spain*
[2]*University of Cambridge, JJ Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
[3]*Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Straße 38, D-01187 Dresden, Germany*
[4]*Max Planck Institute for Chemical Physics of Solids, Nöthnitzer Straße 40, D-01187 Dresden, Germany*
[5]*CFIF, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais, 1049-001 Lisboa, Portugal*
[6]*TCM Group, Cavendish Laboratory, University of Cambridge, JJ Thomson Avenue, Cambridge CB3 0HE, United Kingdom*
(Received 8 December 2013; revised manuscript received 10 March 2014; published 18 June 2014)

The topological features of low-dimensional superconductors have created a lot of excitement recently because of their broad range of applications in quantum information and their potential to reveal novel phases of quantum matter. A potential problem for practical applications is the presence of phase slips that break phase coherence. Dissipation in nontopological superconductors suppresses phase slips and can restore long-range order. Here, we investigate the role of dissipation in a topological Josephson junction. We show that the combined effects of topology and dissipation keep phase and antiphase slips strongly correlated so that the device is superconducting even under conditions where a nontopological device would be resistive. The resistive transition occurs at a critical value of the dissipation that is 4 times smaller than that expected for a conventional Josephson junction. We propose that this difference could be employed as a robust experimental signature of topological superconductivity.

DOI: 10.1103/PhysRevLett.112.247001 PACS numbers: 74.50.+r, 03.75.Lm, 74.40.−n

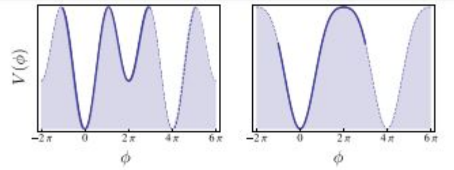


FIG. 1 (color online). Effective potential Eq. (2) for odd parity controlling the phase dynamic of a topological superconducting junction. Case A: $0 < \lambda < 4\mu$ and both a local and a global minimum exist. Case B: $\lambda > 4\mu$ and only a global minimum

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