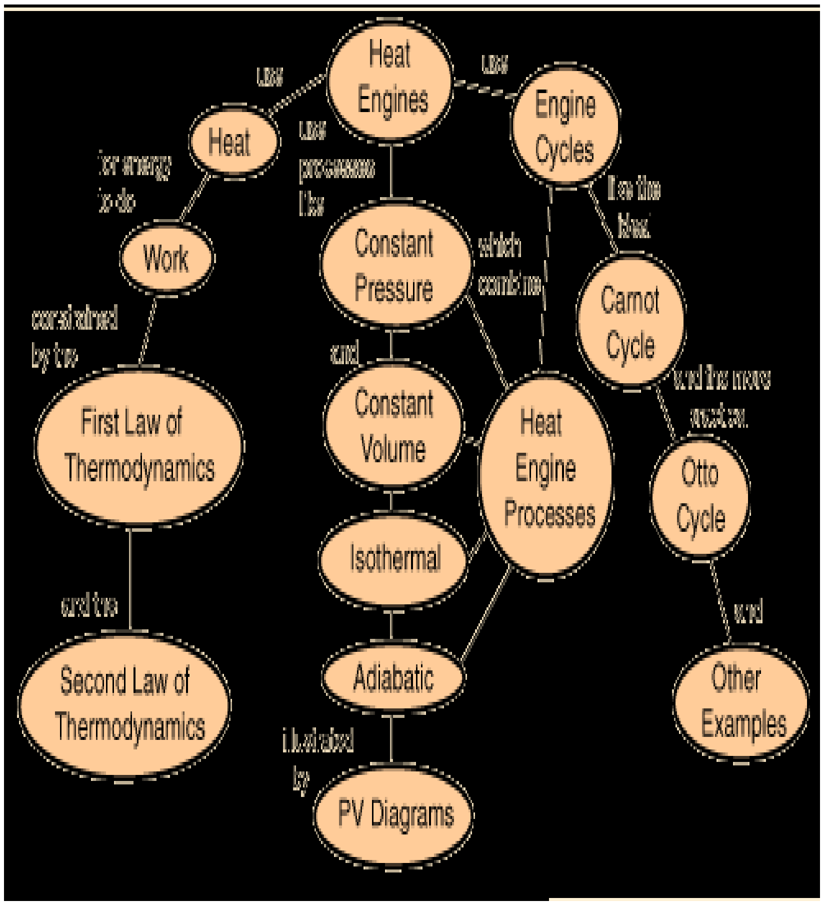
Special Coupled Quantum Heat Engines

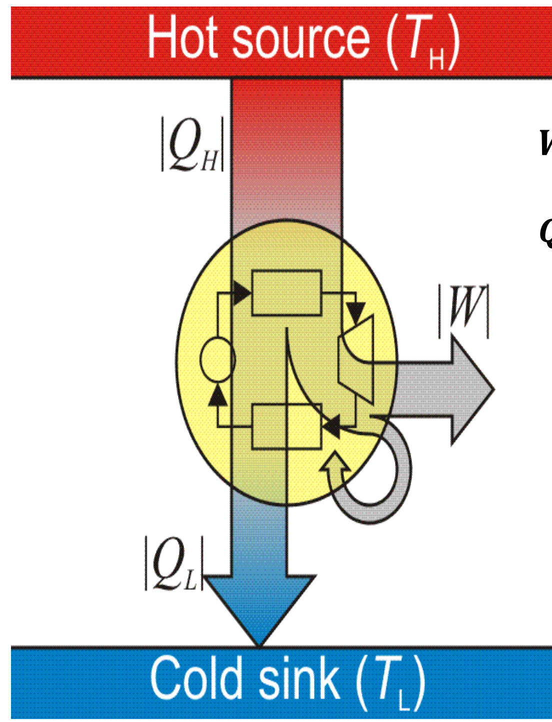
Ferdi ALTINTAŞ

References

[1] H.T. Quan, Y.-X Liu, C.P. Sun and F. Nori, Phys. Rev. E **76** (2007) 031105.
[2] S. Cakmak, D. Turkpence and F. Altintas, Eur. Phys. J. Plus **132** (2017) 554.
[3] F. Altintas and O.E. Mustecaplioglu, Phys. Rev. E **92** (2015) 022142.

The Concept of Heat Engines





$W = Q_H + Q_L$ (1^{st} Law)

$Q_H > -Q_L > 0$ (2^{nd} Law)

Review of Existing Literature

- The introduction of the concept of a QHE - **Three-Level Masers as Heat Engines** by H. Scovil and E. Schulz-Dubois, Phys. Rev. Lett. **2** (1959) 262.

- **Quantum systems as working substances:**

(i) Two- or Multi-level Systems:	**[1]** T.U. Kieu, Phys. Rev. Lett. **93** (2004) 140403 **[2]** H.T. Quan *et al.,* Phys. Rev. E **72** (2005) 056110
(ii) Harmonic Oscillator Systems:	**[1]** B.H. Lin and J.C. Chen, Phys. Rev. E **67** (2003) 046105 **[2**] Y. Rezek and R. Kosloff, New J. Phys. **8** (2006) 83
(iii) Coupled Spin Systems:	**[1]** G. Thomas and R.S. Johal, Phys. Rev. E **83** (2011) 031135 **[2]** T. Zhang *et al.,* Phys. Rev. A **75** (2007) 062102
(iv) CQED systems:	**[1]** M.O. Scully *et al.*, Science **299** (2003) 862 **[2]** H.T. Quan *et al*., Phys. Rev. E **73** (2006) 036122

Quantum Systems

- Discreteness of states
- Quantum Correlations
- Quantum Coherence etc.

→

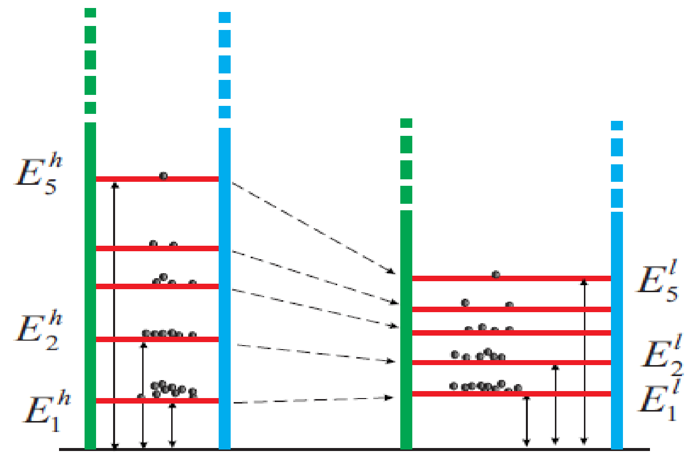
Exotic features

- Surpass the efficiency of a classical Carnot Engine.
- Surpass the maximum limit of the work done by a classical Heat Engine.
- Work extraction from a single heat bath via vanishing quantum coherence.

Quantum 1st Law of Thermodynamics

- The Hamiltonian of a quantum working substance
- $H = \sum_n E_n |n\rangle\langle n|$

  Eigen-energy Eigen-state



- Internal energy

  $U = \langle H \rangle = \sum_n P_n E_n$

  Occupation probabilities

- The infinitesimal change

  $dU = \sum_n (E_n dP_n + P_n dE_n) = đQ + đW$

  → $đQ = \sum_n E_n \, dP_n$ (The heat exchanged corresponds to the change in occupation probabilities.)

  → $đW = \sum_n P_n \, dE_n$ (The work performed corresponds to the change in the energy eigenstates E_n.)

  ! $đQ = T\mathrm{d}S$ is applicable only to the thermal equilibrium case,

  while $đQ = \sum_n E_n \, dP_n$ is general for quantum mechanical systems.

Effective Temperature

What is the temperature operator $\hat{T}$ in QM?

Boltzmann distribution $P_n = \frac{1}{Z} e^{-E_n/k_B T}$

$$\frac{P_n}{P_m} = \frac{e^{-E_n/k_B T}}{e^{-E_m/k_B T}} \longrightarrow k_B T = \frac{E_n - E_m}{lnP_n - lnP_m}$$

Thermodynamical processes

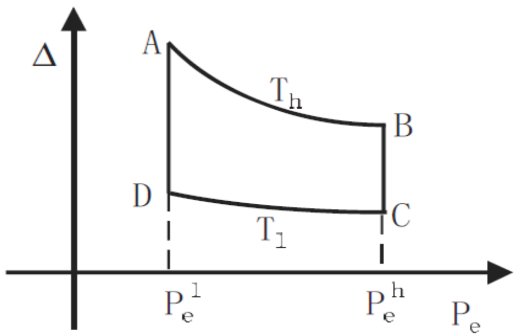
QUANTUM THERMODYNAMIC CYCLES AND QUANTUM HEAT … PHYSICAL REVIEW E **76**, 031105 (2007)

TABLE I. Quantum vs classical thermodynamic processes. Here we use "INV" to indicate the invariance of a thermodynamic quantity and "VAR" to indicate that it varies or changes. U is the internal energy of the working substance; T, P, E_n, P_n are defined in Sec. II. The working substance of the classical thermodynamic processes considered here is the ideal classical gas.

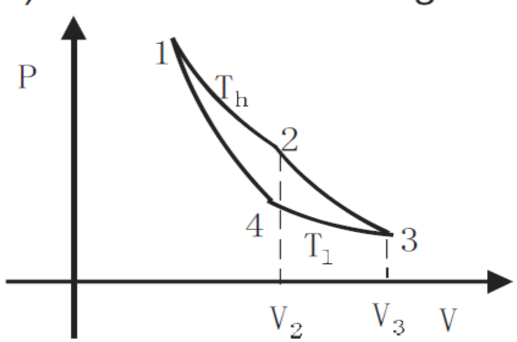
	Isothermal process	Isochoric process	Adiabatic process
Classical	Heat absorbed or released Work done INV: U,T VAR: P,V	Heat absorbed or released No work done INV: V VAR: P,T	No heat exchange Work done VAR: P,T,V
Quantum	Heat absorbed or released Work done INV: T VAR: U,E_n,P_n	Heat absorbed or released No work done INV: E_n VAR: $P_n,T_{\rm eff}$	No heat exchange Work done INV: P_n VAR: E_n, $T_{\rm eff}$

Quantum Carnot Engine (QCE) Cycle

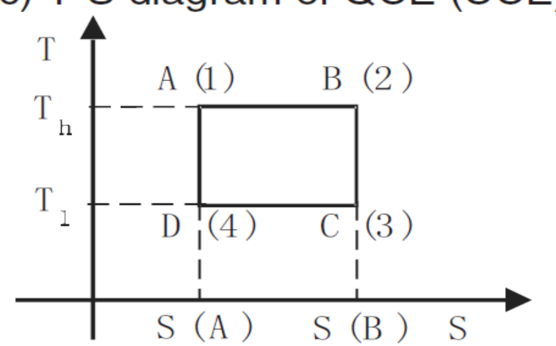
(a) quantum Carnot engine



(b) classical Carnot engine



(c) T-S diagram of QCE (CCE)



A → B (C → D) : quantum **isothermal** processes

B → C (D → A) : quantum **adiabatic** processes

Thermodynamical reversibility of the QCE cycle:

- At point A and point C, the working substance is an effective temperature T(A)=$\mathbf{T_h}$ and T (C)=$\mathbf{T_l}$

- $E_n(B)-E_m(B)=\alpha(E_n(C)-E_m(C))$
  $E_n(A)-E_m(A)=\alpha(E_n(D)-E_m(D))$,

$$\boldsymbol{\alpha = \frac{T_h}{T_l}}$$

(All energy gaps are changed by the **same ratios** in the two quantum adiabatic processes.)

$$Q_{in}^{A\to B} = T_h(S(B) - S(A))$$

$$Q_{out}^{C\to D} = T_l(S(D) - S(C))$$

$$S = -k_B \sum_n P_n ln P_n$$

(Thermodynamical entropy)

1st Law: $\Delta\mathbf{U} = \mathbf{0} \to \boldsymbol{W_C} = \mathbf{Q_{in}^{A\to B}} + \mathbf{Q_{out}^{C\to D}} = (\boldsymbol{T_h} - \boldsymbol{T_l})(\boldsymbol{S}(\boldsymbol{B}) - \boldsymbol{S}(\boldsymbol{A}))$,

$s(A) = S(D)$
$s(B) = S(C)$

- $\eta = \dfrac{\boldsymbol{W_C}}{\boldsymbol{Q_{in}^{A\to B}}} = \mathbf{1} - \dfrac{\boldsymbol{T_l}}{\boldsymbol{T_h}}$

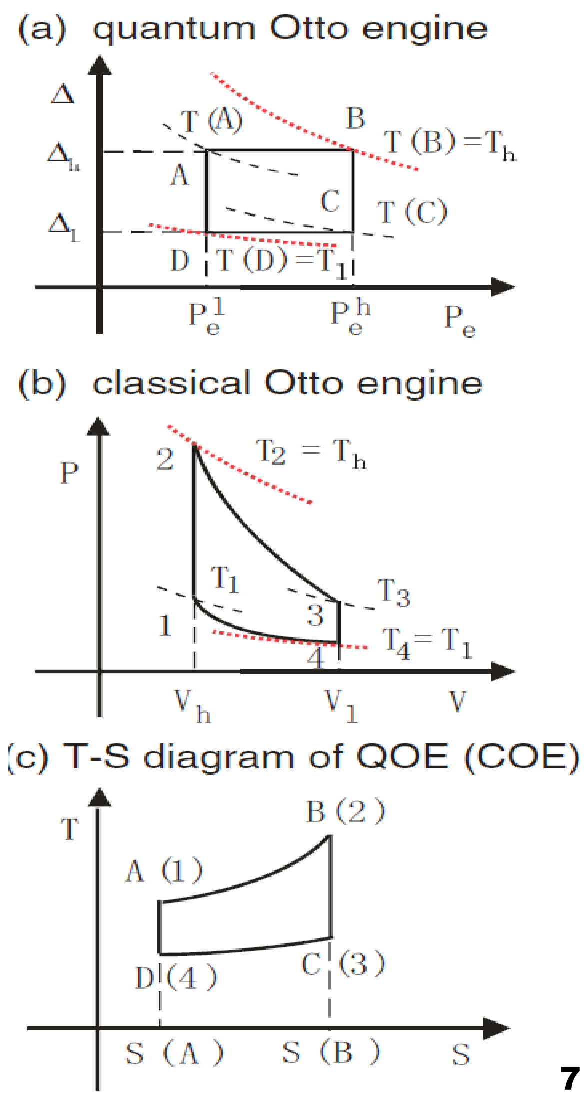
- $PWC(\boldsymbol{W_C} > 0)\ T_h > T_l$

Classical results

Quantum Otto Engine (QOE) Cycle

- A→B (C→D) quantum **isochoric** process
- B→C (D→A) quantum **adiabatic** process
- $Q_{in}^{QIC} = \sum_n E_n^h (P_n(B) - P_n(A))$
- $Q_{out}^{QIC} = \sum_n E_n^l (P_n(D) - P_n(C))$
- $W_O = Q_{in}^{QIC} + Q_{out}^{QIC} = \sum_n (E_n^h - E_n^l)(P_n(B) - P_n(A))$

  $P_n(B) = P_n(C);\ P_n(A) = P_n(D)$
- All energy gaps are changed by the **same** ratios in the quantum adiabatic stages (!!! not necessary !!!).
- $E_n^h - E_m^h = \alpha(E_n^l - E_m^l),\ \alpha \neq \frac{T_h}{T_l}$ (Model independent)
- $\boldsymbol{\eta = \frac{W_O}{Q_{in}^{QIC}} = 1 - \frac{1}{\alpha}}$ (α is an important parameter)
- $PWC(W_O > 0)\ T_h > \alpha\, T_l$



Quantum versus Classical Otto Engines

Claim: The thermal efficiencies of a QOE and its classical counterpart (ideal gas) are equivalent.

Prove: **QOE Cycle**

$E_n^h - E_m^h = \alpha(E_n^l - E_m^l)$; quantum adiabatic theorem

$T(C) = \frac{T_h}{\alpha}$, $T(A) = T_l\alpha$

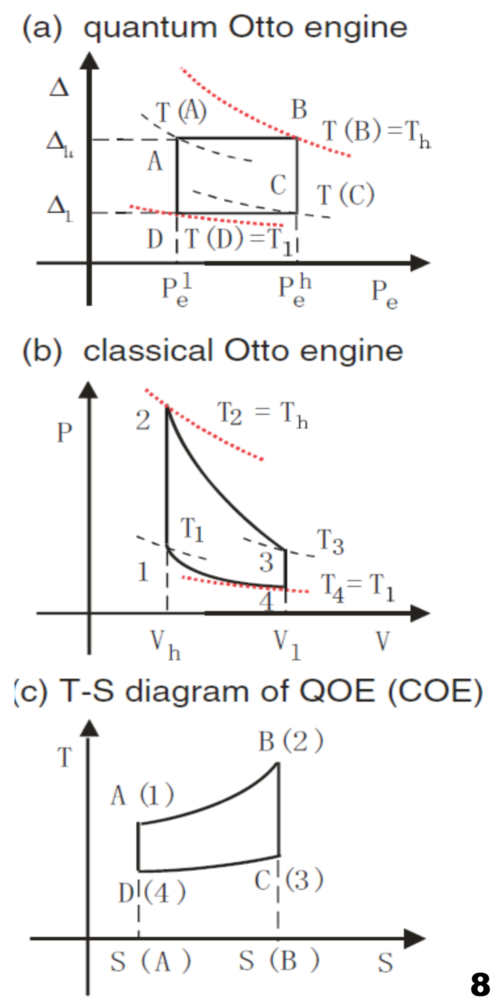
$$\eta = \mathbf{1} - \frac{\mathbf{1}}{\boldsymbol{\alpha}} = \mathbf{1} - \frac{\boldsymbol{T(C)}}{\boldsymbol{T(B)}} = \mathbf{1} - \frac{\boldsymbol{T(D)}}{\boldsymbol{T(A)}}$$

COE Cycle

$$\eta^{CL} = 1 - (\frac{V_h}{V_l})^{\gamma-1}$$

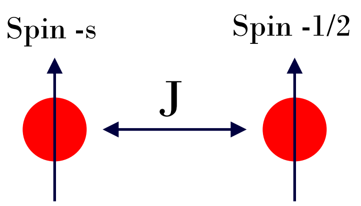
$TV^{\gamma-1}$ = constant in a classical adiabatic process

$$\eta^{CL} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_1} \qquad \Rightarrow \eta = \eta^{CL}$$



Special Coupled QOE and QCE Cycles

Working substance



$$H = \omega\left(S_Z^A + S_Z^B\right) + J(S_x^A S_x^B + S_y^A S_y^B + S_Z^A S_Z^B)$$

$$S_\alpha^A = S_\alpha \otimes \mathbf{1}_2$$

$$S_\alpha^B = \mathbf{1}_{(2s+1)} \otimes S_\alpha$$

$$\left[S_\alpha, S_\beta\right] = \mathbb{i}\boldsymbol{\varepsilon}_{\alpha\beta\gamma} S_\gamma$$

ω : Bohr frequencies

$J(> 0)$: Anti-ferromagnetic coupling strength

Quantum Otto Engine Cycle

- Adiabatic changes: simultaneous change in ω and J
  $(\omega_h \rightarrow \omega_l \rightarrow \omega_h)$ and $(J_h \rightarrow J_l \rightarrow J_h)$
- $\frac{J_h}{\omega_h} = \frac{J_l}{\omega_l} = r$; r=0 uncoupled case
  r>0 coupled case

Relative coupling strength

- For any spin-s, $E_n^h - E_m^h = \alpha(E_n^l - E_m^l)$

$$\alpha = \frac{\omega_h}{\omega_l}$$

- $\eta = 1 - \frac{\omega_l}{\omega_h}$

- $PWC,\ T_h > \frac{\omega_h}{\omega_l} T_l$

Same as a **qubit** as a working subtance

The role of spin-s and the quantum interactions on the relative work output.

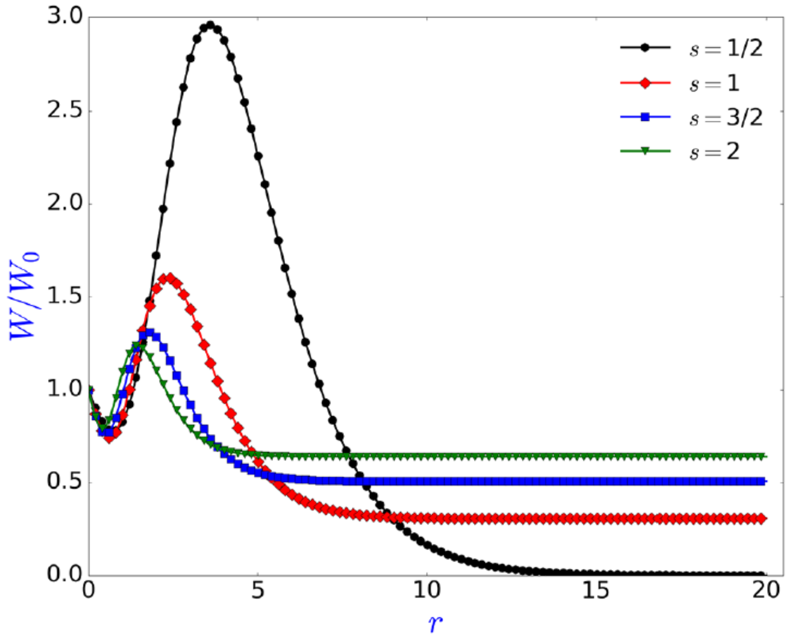


Fig. 1. (Otto Cycle) The work obtained from the special coupled heat engine, *W*, divided by the corresponding work from the uncoupled $(r = 0)$ one, W_0, as a function of the relative coupling strength r for the ratios $T_h/T_l = 2.0$, $\omega_h/\omega_l = 1.5$ and the spin-*s* values, $s = 1/2, 1, 3/2, 2$. The thermal efficiency and the Carnot limit are given $\eta = 1/3$ and $\eta_c = 0.5$, respectively.

- The thermodynamics at deep strong coupling regime, i.e. $\mathbf{J} \to \infty$

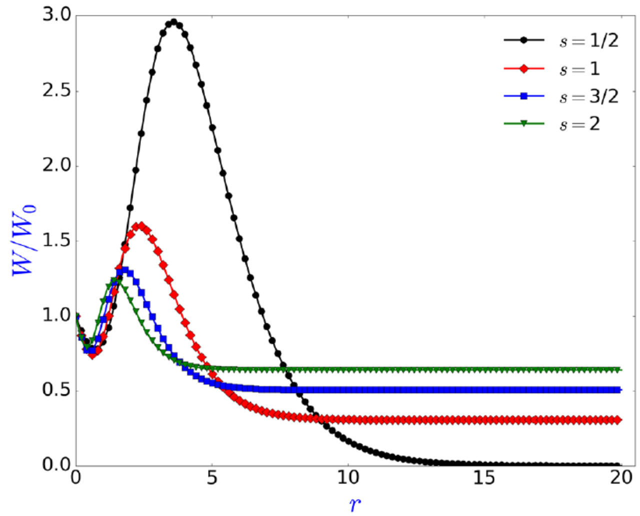
- Analyses: Thermodynamical quantities are **invariant** under uniform energy shifts

$$H = \sum_n (E_n + \delta)\,|n\rangle\langle n|$$

Adding δ to all energy levels is irrelevant.

Therefore, when $\mathrm{J} \to \infty$, (s, ½) model system can be mapped into a multilevel system with energy spectrum:

$$\{0, \omega, 2\omega, \ldots, (2s-1)\omega\}$$



where $\eta = 0$ for $s = 1/2$, while $\eta = 1 - \frac{\omega_l}{\omega_h}$ for $s > 1/2$.

The role of spin-s on the maximum relative work output

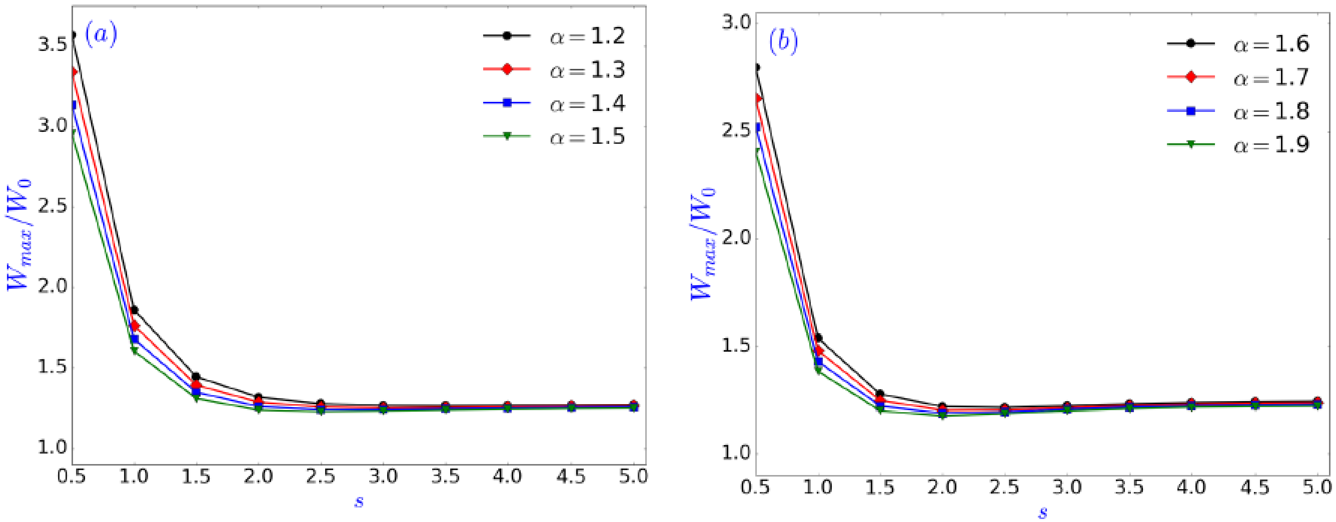


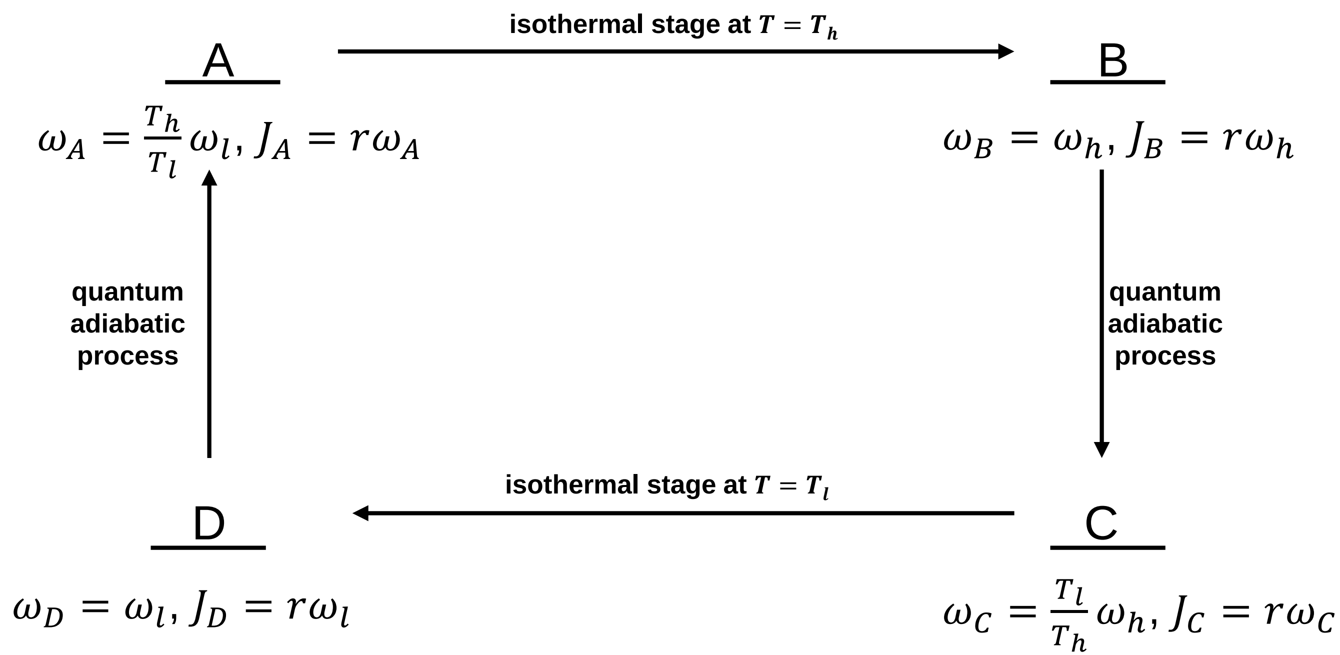
Fig. 2. (Otto Cyle) The maximum work obtained from the special coupled heat engine, W_{max}, divided by the work obtained from the uncoupled $(r = 0)$ one, W_0, as a function of the value of the spin-*s* for the ratio $T_h/T_l = 2.0$, and for different $\alpha = \frac{\omega_h}{\omega_l}$ values. Note that the thermal efficiency depends only on α $(\eta = 1 - 1/\alpha)$ and the Carnot limit is $\eta_C = 0.5$.

Results:

- $\frac{W_{max}}{W_0} > 1$ $\rightarrow$ quantum coupling enhanced work output.
- $\frac{W_{max}}{W_0}$ decreases monotonically when s increases.
- As α increases, $\eta = 1 - \frac{1}{\alpha}$ increases while $\frac{W_{max}}{W_0}$ decreases.

Special Coupled Quantum Carnot Engine

- (s, ½) Heisenberg XXX system as a working substance



- Thermodynamically reversible

$$\mathrm{E_n(A) - E_m(A) = \frac{T_h}{T_l}(E_n(D) - E_m(D))}$$

$$\mathrm{E_n(B) - E_m(B) = \frac{T_h}{T_l}(E_n(C) - E_m(C))}$$

- $\eta_C = 1 - \frac{T_l}{T_h}$
- $PWC(W > 0)\ \ T_h > T_l$

‘Classical results’

The role of spin-s and quantum interaction on the relative work output.

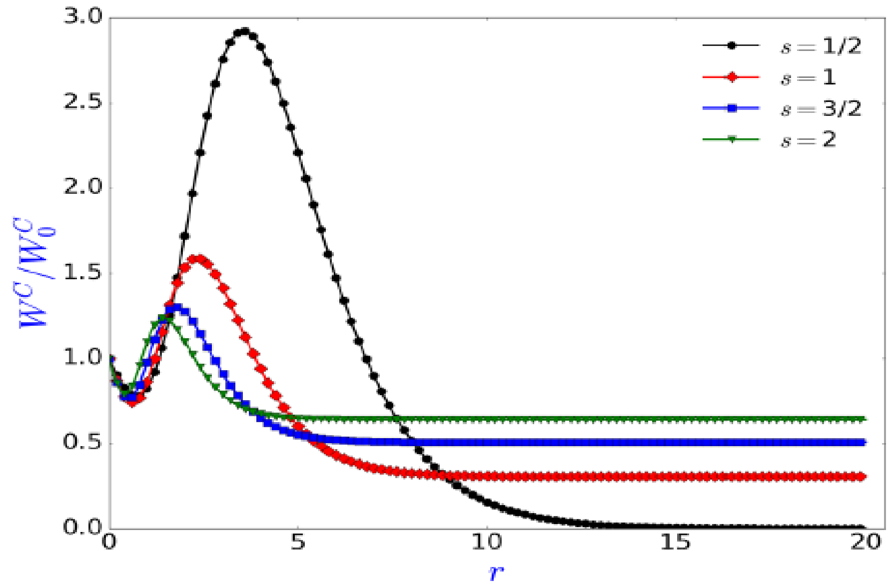


Fig. 3. (Carnot Cycle) The work obtained from the special coupled QCE cycle, W_C, divided by the corresponding work from the uncoupled $(r = 0)$ one, W_0^C , as a function of the relative coupling strength *r* for the ratios T_h/T_l=2.0, $\omega_h/\omega_l = 1.5$ and the spin-*s* values, $s = 1/2, 1, 3/2, 2$. The classical Carnot efficiency is $\eta_C = 0.5$.

Active research interest for QHEs

1. Role of quantum properties in the working substance (quantum coherence, interactions and entanglement) on the work output and thermal efficiency.

   [1] Phys. Rev. E **90**, 032102 (2014)

   [2] Phys. Rev. E **83**, 031135 (2011).

2. Local thermodynamics and its relation with the global one; work output is not an extensive quantity.

   [1] Phys. Rev. E **83**, 031135 (2011).

   [2] Phys. Rev. E **92**, 022142 (2015).

3. Use of quantum heat baths (entangled or quantum coherent or squeezed or non-Markovian); classical Carnot efficiency is not the upper bound.

   [1] EPL **88**, 50003 (2009).

   [2] Science **299**, 862 (2003).

4. Role of the time-dependent changes in adiabatic branches; source of non-adiabatic dissipation; inner friction (a fully quantum mechanical phenomena)

   [1] Eur. Phys. J. D **71**, 75 (2017).

   [2] New J. Phys. **17**, 075007 (2015).

Thank you

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