Energetic Cost of Information Processing at the Quantum Precipice:

A Physical-Information-Theoretic Approach

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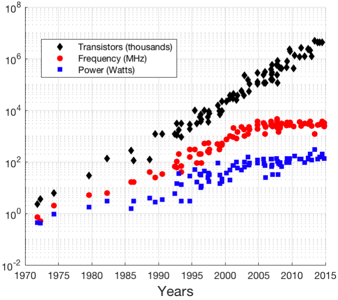
Research Motivation

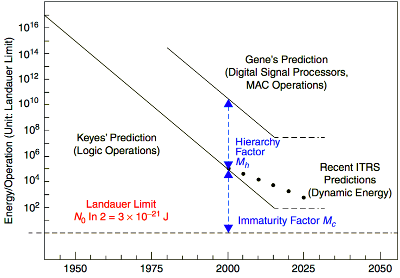


45-year trend in microprocessors.[1]

- More than Moore: The future of computing and **rebooting** the trend.
- Numerous factors play a role in the future of computing technologies, however, **energy dissipation** is a critical one.

[1] Original data up to the year 2010 collected by M. Horowitz, F. Labonte, O. Shacham, K. Olukotun, L. Hammond, and C. Batten, data between 2010-2015 collected by K. Rupp.

Research Motivation



Energy per operation w.r.t. the Landauer limit at room temperature for transistor-based devices.[2]

- Information-theoretic energy dissipation may impose practical limitations on ultimate capabilities of emerging nano-computing circuits.
- Fundamental lower bounds become more and more technologically relevant.

[2] Aarne Mämmelä and Antti Anttonen, "Why Will Computing Power Need Particular Attention in Future Wireless Devices?," *IEEE Circuits and Systems Magazine*, pp. 13-26, February 13, 2017. DOI: 10.1109/MCAS.2016.2642679

Research Objective

We aim to:

- Develop a methodology for determining **fundamental lower bounds** on the dissipative cost of computation in new and unfamiliar **nanocomputing paradigms**.
- Evaluate **energy dissipation** under best case assumption for **concrete nanocircuits and nanoprocessors.**
- Utilize these lower bounds as a **fundamental efficiency assessment tool** for complex circuits as efficiencies approach fundamental limits.

Overview

1. Background
2. Methodology
3. Classical Information Processing Applications
4. Quantum Information Processing Applications
5. Conclusions

Background

- Information is tied to a physical representation.
- "Any logically irreversible manipulation of information, such as the erasure of a bit or the merging of two computation paths, must be accompanied by a corresponding entropy increase in non-information bearing degrees of freedom of the information processing apparatus or its environment."[3]

$$\Delta S \geq k_B \ln(2) \Delta I_{er} \tag{1}$$

$$\Delta E \geq k_B \ln(2) T \Delta I_{er} \tag{2}$$

Here, k_B is Boltzmann constant, T is temperature, and ΔI_{er} is the amount of information erased from a physical system.

[3] C. H. Bennett, "Notes on Landauer's principle, reversible computation, and Maxwell's Demon," *Studies in History and Philosophy of Modern Physics*, vol. 34, pp. 501-510, 2003.

Landauer's Principle (LP) and Its Practical Implications

- Landauer's Principle is an **idealization**.
- Information erasure defined in terms of **self-entropy** may not capture resulting dissipation accurately.
- LP is **paradigm-independent**; few attempts have been made to evaluate the fundamental minimum cost of computation in concrete, nontrivial computing scenarios.
- In the last five years, 200+ papers are published on LP and almost one-third of them are experimental testing of the principle.[4]

[4] Based on Google Scholar search, 30/01/2018.

Referential Approach

- Information erasure is regarded as "**loss of correlation** between the state of an erasable quantum system and that of an enduring referent system holding classical information."[5]
- Fundamental bounds for single-shot quantum L-machines can be obtained using this approach:
  - Lower bound on the average expected energy of the environment in terms of information loss and average entropy change of the representative states during computation

$$\left\langle \Delta \left\langle E_i^{\mathscr{E}} \right\rangle \right\rangle \geq k_B \ln(2) T \left\{ \Delta I_{er} - \left\langle \Delta S_i^{\mathcal{S}} \right\rangle \right\} . \qquad (3)$$

    Here, $\left\langle \Delta S_i^{S} \right\rangle$ is the average entropy reduction in the information bearing system, $\mathcal{S}$.[6]

[5] N. G. Anderson, "Information erasure in quantum systems," *Phys. Lett. A* , vol. 372, pp. 5552-5555, 2008.

[6] N. G. Anderson, "On the physical implementation of logical transformations: Generalized L-machines," *Theoretical Computer Science*, vol. 411, pp. 4179-4199, 2010.

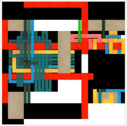
METHODOLOGY

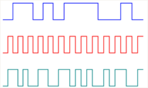
FUELCOST

An evolving methodology based on physical information theory that is designed for systematic determination of FUndamental energy Efficiency Limits of computation in complex COmputing STructures.

Nature of our Methodology

Nanocircuit & Clocking Scheme

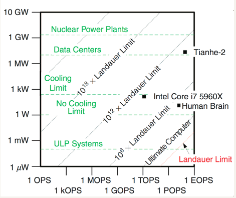




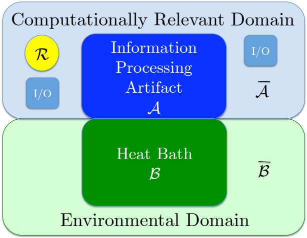
Methodology

1. Abstraction
   - Physical Abstraction
   - Process Abstraction
2. Analysis
   - Operational Decomposition
   - Cost Analysis

Fundamental Limits



Methodology: Physical Abstraction



Abstraction	Physical Circuit
$\mathcal{A}$	Information Processing Artifact
$\bar{\mathcal{A}}$	Supporting Computational Subsystems
$\mathcal{B}$	Heat Bath
$\bar{\mathcal{B}}$	External Heat Reservoir

- Referent: a physical system holding input data: $\hat{\rho}^{R_\eta} = \sum_{i=1}^{N} p_i \left| x_i^{R_\eta} \right\rangle \left\langle x_i^{R_\eta} \right|$ where the are orthogonal pure states encoding the inputs $\{x_i\}$ and p_i is the i^{th} input probability.
- Initial state of the globally closed isolated universe: $\hat{\rho}_0 = \left(\sum_{i=1}^{N} p_i \hat{\rho}_i^{R_\eta} \right) \otimes \hat{\rho}^{A_k} \otimes \hat{\rho}_i^{\bar{A}_k} \otimes \hat{\rho}^{\mathscr{B}} \otimes \hat{\rho}^{\bar{\mathscr{B}}}$

- The globally closed system **evolves unitarily** according to Schrödinger equation.
- System is **drawn away from equilibrium during control** operations, ϕ, and **restored during rethermalization.**
- **State transformations** are obtained for each control and restoration operation.
- All the essential **functional features are captured** based on the underlying computational strategy.[7]

[7] İ. Ercan and N. G. Anderson, "Heat Dissipation in Nanocomputing: Lower Bounds from Physical Information Theory," *IEEE Transactions on Nanotechnology*, Vol. 12, Issue 6, pp. 1047 - 1060, 2013.

Methodology: Analysis

Operational Decomposition

- Clocking
  - Clock zones, $C(u)$, and subzones, $C_l(u)$.
  - Clock steps $\psi_v : \{C(u); \phi_t\}$.
  - Clock cycle $\Phi = \psi_1 \psi_2 \psi_3 \ldots$.
- Computation
  - Computational step c_k.
  - Computational cycle $\Gamma^\eta = c_1 c_2 c_3 \ldots$

Cost Analysis

- Information dynamics: Data zones, $D(c_k)$, and subzones $D_w(c_k)$.
- Dissipation bounds

$$\Delta \langle E \rangle = \sum_{k=1}^{K} \Delta \langle E \rangle_k = \sum_{k=1}^{K} \left[\sum_{w \in \{k-1\}} \Delta \langle E \rangle_{k-1}^{w} \right] \quad (4)$$

Classical Information Processing Applications

Highlights from Earlier Results

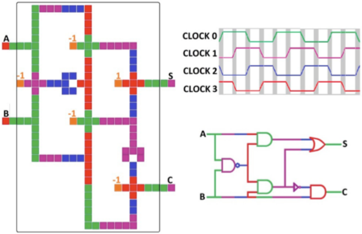
Applications via prominent nanocircuit proposals, such as Quantum Dot Cellular Automata (QCA), Nanoapplication Specific Integrated Circuits (NASICs) and CMOS techonology illustrate that:

- The **unavoidable cost of information processing** significantly depends on the **details of underlying computing strategy**.
- **Accurate level of granularity** is of key importance for isolating irreversibility in a circuit.
- In transistor-based circuits, **particle supply costs dominate the bound** in transistor-based paradigms, exceeding the cost of the irreversible logic operations.

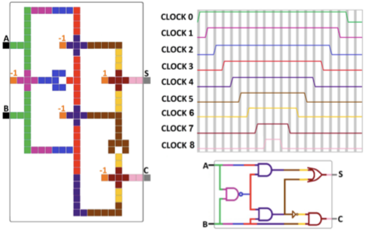
Non-Transistor-Based Application: QCA Half Adder

- Layout of the QCA half adder circuit along with associated logic and clocking diagrams[8]

Landauer Clocking



Bennett Clocking



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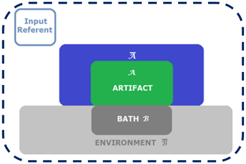
[6] İ. Ercan and N. Anderson, "Heat Dissipation in Nanocomputing: Theory and QCA Application," *Proceedings of the 11th IEEE Conference on Nanotechnology* (IEEE NANO, 2011), pp.1289-1294, 2011. (Best Paper Award)

QCA Half Adder Abstraction

Physical Abstraction



Abstraction	Physical Circuit
$\mathcal{A}$	QCA Circuit
$\bar{\mathcal{A}}$	External registers, Adjacent circuit stages
$\mathcal{B}$	Substrate in thermal contact with the artifact, at T
$\bar{\mathcal{B}}$	External Heat Reservoir

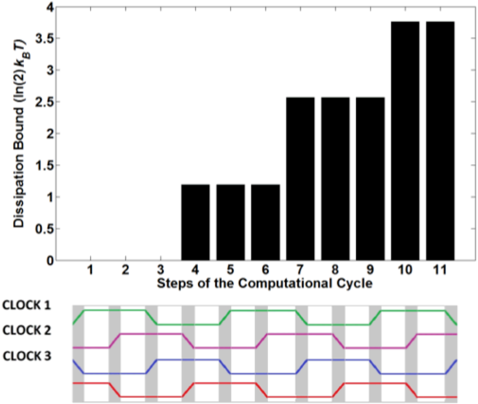
- $\mathscr{R}$: a physical system holding input data.

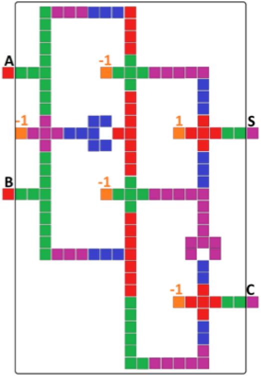
Process Abstraction

- Globally closed system evolving via Schrödinger equation.
- System is drawn away from equilibrium during clock operations: Switch, ϕ_1, Hold, ϕ_2, Release, ϕ_3, and Relax , ϕ_4, and restored during rethermalization.

QCA Half Adder Analysis: Landauer Clocking

Lower bound on the cumulative dissipative cost for one computational cycle of the Landauer-clocked QCA half circuit





QCA Half Adder Analysis: Landauer Clocking

- Bound for the a given computational step k

$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_k \geq \sum_{C_w^k} -k_B T \ln(2) \left(\Delta I^{C_w^k D_w^k} \right)$$

- Fundamental Dissipation Bound

$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_{TOT} \geq 3.76 k_B T \ln(2)$$

Reflects unavoidable costs of underlying computation strategy under Landauer clocking –independent of material parameters, device and circuit dimensions.

QCA Half Adder Analysis: Bennett Clocking

- Proposed by Lent and co-workers as a way to implement reversible computation in QCA circuits and improve power efficiency.[9]
- Dissipation-free sequences of computations are possible but only by reversible unloading the input data at the end of each cycle.[10]
- Multiple Bennett-clocked stages of QCA requires input data to be erased at the end of computational cycle.

[9] C.S. Lent, M. Liu and Y. Lu, "Bennett clocking of quantum-dot cellular automata and the limits to binary logic scaling," *Nanotechnology*, Vol. 17, pp. 4240-4251, 2006.

[10] N. G. Anderson, "Reversible Computation via Bennett Clocking in QCA Circuits: Input-Output Requirements," *Proceedings of the of the 2009 International Workshop on Quantum-Dot Cellular Automata*, pp. 12-13 (2009).

QCA Half Adder Analysis: Bennett Clocking

- Unavoidable dissipative cost of each computation cycle is that of erasing two bits of input information at the last step of computation.
- Fundamental Dissipation Bound

$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_{TOT} \geq 2k_B T \ln(2)$$

Fundamental lower bound on the dissipative cost per computational cycle for Bennett clocking is smaller than the corresponding bound for Landauer clocking by a factor of 0.53 (=2/3.76).

Remarks on QCA Half Adder Applications

- Circuits operated under Landauer clocking dissipates more heat per computational cycle.
- Under Bennett clocking circuits need to be operated at a higher rate to compensate for the increased inherent latency.
- Under Bennett clocking the areal power dissipation is less than the corresponding bound for Landauer clocking.

Fundamental Bounds for Transistor-based Circuits

- For transistor-based combinational circuits where the physical state evolution and logic state evolution has $1-1$ correspondence, the fundamental lower bound on dissipation in a circuit C_k
  - For a given computational step k

$$\Delta\left\langle E^{\mathcal{B}}\right\rangle_k \geq -k_B T \ln(2)\left(\Delta I^{\mathcal{R}_\eta C_k} + \left\langle \Delta S_i^{C_k}\right\rangle + \left\langle \Delta S_i^{\mathcal{S}}\right\rangle_k + \left\langle \Delta S_i^{\mathcal{D}}\right\rangle_k\right)$$

  - For the complete computational cycle

$$\Delta\left\langle E^{\mathcal{B}}\right\rangle_{TOT} \geq k_B T \ln(2)\Delta I + f q V_{DD}\Delta N$$

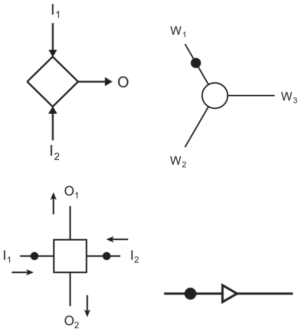
  here, f is a fraction of the energy invested in the circuit by the source, $\mathcal{S}$, and drain, $\mathcal{D}$, and ΔN is the number of charges transferred from $\mathcal{S}$ to $\mathcal{D}$.

Brownian Circuits[11]

- A novel emerging computing paradigm that exploits fluctuations by finding computational paths in circuits through a random search mechanism.
- Brownian circuits can exploit fluctuations to increase the efficiency of information processing.
- Signals are represented in terms of tokens, propagate randomly and are driven by local transition rules.

[11] F. Peper, J. Lee, J. Carmona, J. Cortadella, and K. Morita, "Brownian Circuits: Fundamentals" *ACM Journal on Emerging Technologies in Computing Systems*, Vol. 9, No. 1, Article 3, 2013.

Merge, Hub, Conservative Join (CJoin), and Ratchet are defined as primitives of a universal class of Brownian circuits.[12]



[12] J. Lee, F. Peper, S. D. Cotofana, M. Naruse, M. Ohtsu, T. Kawazoe, Y. Takashi, T. Shimokawa, L. Kish, and T. Kubota, "Brownian Circuits: Designs" *Int. Journ. of Unconventional Computing*, Vol. 12, pp. 341–362, 2016.

Brownian NAND and Half-Adder Circuits

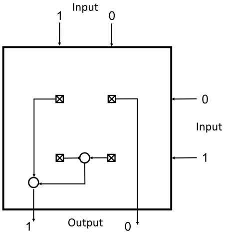


Figure: Brownian NAND constructed from a 2 × 2-CJoin and two Hubs.

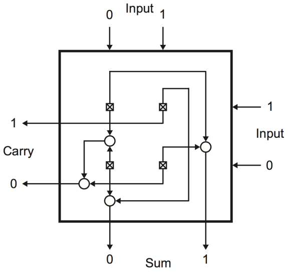


Figure: Brownian half-adder constructed from a 2 × 2-CJoin and four Hubs.[11]

Realization of Brownian Circuits

- For a technology to be suitable for implementations of Brownian circuits, it is necessary that (a) it supports representation of signals that is token-based and that (b) are subject to fluctuations.
- Potential candidates for realization of Brownian circuits are:
  - Spintronics, in which information is encoded by the spins of electrons.
  - Nanophotonics, where a token would represent an electron hole pair, which results from the absorption of a photon in a semiconductor.
  - Single Electron Tunneling (SET) circuits which use tunneling of electric charge as the underlying operating mechanism.

Single Electron Tunneling (SET) Transistors

- A SET transistor uses controlled electron tunneling to amplify current.
- The fundamental element in a SET circuit is a tunneling junction, which is an thin layer with high electrical resistance separating two Coulomb islands.

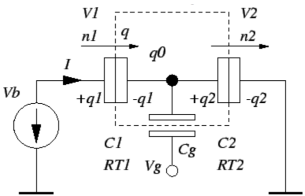


Figure: SET transistor.[13]

[13] C. Wasshuber, "About Single-Electron Devices and Circuits," *Ph.D. Dissertation* Vienna University of Technology, 1997.

SET Realization of Brownian Circuits

- In order to realize Brownian circuits in SET transistors, we need to use token-based character of electrons by making the tunneling resistance sufficiently high.
- The resistance of tunneling junctions R_j needs to be much greater than $h/q^2 = 25.8k\Omega$, which is commonly chosen as $100k\Omega$ in literature.
- In order to have controlled-tunneling, the charging energy, E_c, needs to be sufficiently high, i.e. $E_c = \frac{q^2}{2C} \gg k_B T$.
- This condition implies an upper bound on the size of the quantum dot, i.e. the capacitance of a Coulomb island cannot exceed $926aF$ at $1K$.

SET Realization of a Hub and CJoin

Single Electron Tunneling (SET) realization of a Hub and CJoin:

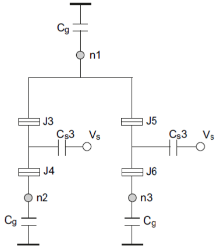


Figure: SET-based Hub.

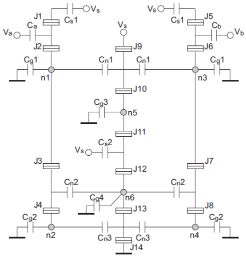
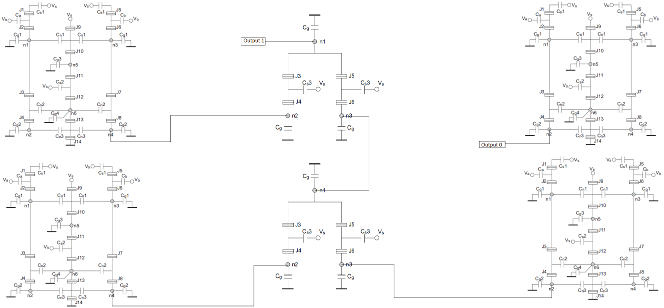


Figure: SET-based CJoin.

SET Realization of a Brownian NAND Circuit

Single Electron Tunneling (SET) realization of a Brownian NAND circuit:



Abstraction of SET-based Brownian Half-Adder

Physical Abstraction

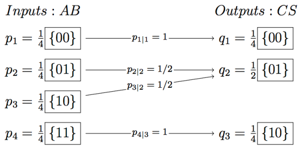
- The referent, $\mathscr{R}$, is a physical system holding input data.
- Source, $\mathscr{S}$, and drain $\mathscr{D}$ are regarded as idealized Fermi gases that are nominally at T.
- The circuit, $\mathscr{C}$, is composed of quantum dots, tunnel junctions and capacitors.
- The bath, $\mathscr{B}$, is responsible from rethermalizing the $\mathscr{S}$, $\mathscr{C}$, and $\mathscr{D}$.
- The external reservoir, $\hat{\mathscr{B}}$, recharges $\mathscr{S}$ and $\mathscr{D}$.

Process Abstraction

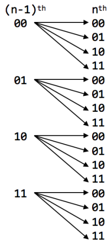
- Globally closed system evolving via Schrödinger equation.
- Brownian circuits are asynchronous, dual-rail encoding is needed to represent logic values.
- System is drawn away from equilibrium during computation and then restored during rethermalization.

Logic-state transformations

- The logic-state transformations for a Half-Adder can be obtained by input to output mapping and by considering that the $(n-1)^{th}$ input is overwritten by the next input, n^{th}.



Input to output logic-state mapping of a Half-Adder circuit.



Logic-state transformation of a Half-Adder circuit by overwriting of information.

- Brownian circuits are not reset between computation, and the information is erased by overwriting with a new input.

Preliminary Results on Fundamental Limits

The fundamental lower bound on energy dissipation of one input, averaged over all possible logic-state transformations, for equiprobable inputs[14]

- For the SET-based Brownian NAND gate

$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_{TOT} \geq k_B T \ln(2) + f q V_{DD} \quad (5)$$

- For the SET-based Brownian Half-Adder

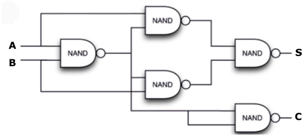
$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_{TOT} \geq k_B T \ln(2) + f 2 q V_{DD} \quad (6)$$

Here, f is a fraction of the energy invested in the artifact by $\mathcal{SD}$.

[14] İ. Ercan and E. Suyabatmaz, "Fundamental Energy Limits of SET-Based Brownian NAND and Half-Adder Circuits" Submitted to *European Physical Journal B* on 30/10/2017.

Comparison with CMOS NAND-based Half-Adder

Consider a hald adder with five NAND gates where each NAND gate can be implemented in CMOS by using 2 NMOS and 2 PMOS transistors.



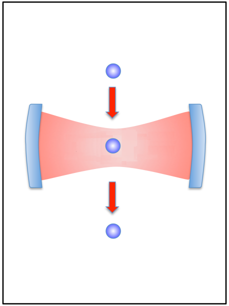
Fundamental lower bound on energy dissipation of one input averaged over all logic-state transformations

$$\Delta \left\langle E^{\mathscr{B}} \right\rangle_{TOT} \geq k_B T \ln(2) + f\, 3.75 q V_{DD}$$

Note that this inequality reflects unavoidable costs of underlying computation strategy – independent of material parameters, device and circuit dimensions.

Quantum Information Processing Applications

Single-atom injected into a microwave cavity can represent a qubit.[15]



The single atom-cavity interaction is described by the Tavis–Cummings model.[16]

$$
\begin{aligned}
H_{TC} &= H_a + H_c + H_{int} \qquad (7)\\
H_a &= \frac{\hbar\omega_a}{2}\sigma^z\\
H_c &= \hbar\omega_c a^\dagger a\\
H_{int} &= \hbar g\left(a\sigma^+ + a^\dagger\sigma^-\right)
\end{aligned}
$$

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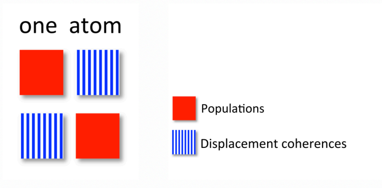
[16] Wolfgang P. Schleich, Herbert Walther, *Elements of Quantum Information*, John Wiley and Sons, Jun 27, 2007.

[16] C. B. Dağ, W. Niedenzu, O. E. Müstecaplıoğlu, and G. Kurizki, "Multiatom Quantum Coherences in Micromasers as Fuel for Thermal and Nonthermal Machines," *Entropy*, 18, 244; doi:10.3390/e18070244 2016.

The master equation and the density matrix of the single atom representing the role of the displacement coherences with respect to the cavity-field evolution are given as:

$$\dot{\rho} \approx -i[H_{eff}, \rho] + \mathbb{L}_{s\rho} + \mathbb{L}_{\rho}$$
$$H_{eff} = pg\tau\left(\lambda a^{\dagger} + \lambda^{*} a\right)$$



Population of the ground, $|g\rangle$, and excited, $|e\rangle$, -states are used to represent the two possible bit values

$$|\psi\rangle = c_1 |g\rangle + c_2 |e\rangle$$

where c_1 and c_2 are any complex numbers satisfying $|c_1|^2 + |c_2|^2 = 1$, i.e. $|\psi\rangle$ is normalized.

Abstraction of Single-Atom Cavity System

Physical Abstraction

- The referent, $\mathscr{R}$, is the initial state of the atom –a laser pulse interaction with the input qubit.
- The artifact, $\mathscr{A}$, is the atom –single qubit memory.
- The cavity $\mathscr{C}$ serves as the bath.
- The external reservoir, $\bar{\mathscr{B}}$ responsible from rethermalizing the complete system.

Process Abstraction

- Globally closed system evolving via Schrödinger equation.
- The atom exchanges heat with the cavity and changes its state.
- System is drawn away from equilibrium during memory erasure and then restored during rethermalization.

Energy Dissipation in the Micromaser System

We assume that the atom is initially in excited-state and the field is in thermal equilibrium at temperature T:

$$\rho^{\mathscr{A}}(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

$$\rho^{\mathscr{C}}(0) = \sum_{n=1}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} |n\rangle\langle n|,$$

where $\bar{n} = 1/(\exp(\hbar\omega/k_b T) - 1)$

The energy loss due to thermal interaction with the cavity is[17]

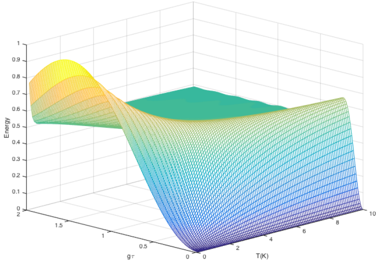
$$Q = \langle \mathrm{H}_q \rangle(\tau) - \langle \mathrm{H}_q \rangle(0) = \frac{\hbar\omega}{2} - \frac{\hbar\omega}{2} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{(1+\bar{n})^{n+1}} \cos(2g\tau\sqrt{n+1}).$$

[17] E. Solano-Carrillo, "Entropy production and thermalization in the one-atom maser," *Phys. Rev. E*, 062116 2016.

The fundamental lower bound on energy dissipation as a result of the single qubit memory erasure due to interaction with cavity can be represented as

$$\Delta E \geq k_B T \ln(2) \Delta S + \Delta Q$$

At resonance frequency $\omega = 5\text{GHz}$, with interaction strength $g\tau = [0, 2]$ and $T = [0, 10]K$.[18]



[18] The author would like to thank Koç University Physics student Angswar Manatuly for calculations.

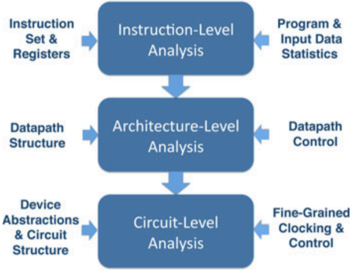
Conclusions

- **Particle supply cost dominates** the fundamental lower bound bound in transistor-based circuits, far exceeding the cost of the irreversible logic operations.
- Implementations based on **SET transistors promise a lower fundamental limits** as compared to other transistor-based circuits such as NASICs.
- Area required for Brownian circuits is small despite the dual rail encoding, however, time required for computation is long, therefore **power dissipation may be relatively high**.
- The manufacturing techniques proposed for **SET transistors allow aggressive scaling** that can mean higher performance, density, and power efficiency that can go far beyond the performance of CMOS technology.

Fundamental lower bounds can also be obtained in architecture- and instruction-level of computing structures.

Certain constraints are imposed in each level by

- physical laws (ILA)
- hardware organization and control (ALA)
- internal circuit structure and operation (CLA).



Hierarchical levels of analysis.[19]

[19] N. G. Anderson, İ. Ercan, N. Ganesh, "Toward Nanoprocessor Thermodynamics," *IEEE Transactions on Nanotechnology*, Vol. 12 issue 6, 2013.

On Quantum Information Processing Applications

- The approach we developed can be generalized to obtained fundamental energy dissipation bounds on quantum information processing.
- Illustrative example we employ here shows that there may be unavoidable thermal energy cost associated with erasure of information in single-atom cavity interaction.
- For high interaction strength, ΔQ can be order of magnitude larger than $k_B T \ln(2) \Delta S$.

Future Directions

- Electronic **circuits with higher complexity** and their **nanoprocessor** applications will be analyzed and compared to simulation results.
- An analysis on **time and energy trade off** will be performed to obtain power dissipation limits.
- Applications to **micro-ring resonator based photonic circuits** will be explored using Binary Decision Diagrams.[20]
- **Quantum information processing applications** will be studied in circuit level and algorithm level.

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[20] Support from MIT International Science and Technology Initiatives Seed Fund, 2018.

Final Remarks

- The dominating factors in this **minimum cost of computation may vary for different paradigms** and must be studied separately.
- Fundamental bounds gives us the **physical possibility** that specified performance targets can be met **under best case assumptions**,
- The **viability** of any proposed emerging nanocomputing technology **hinges on a mixture of factors** such many physical, technological, and economic, etc.
- **Our methodology can serve as a litmus test** in nanocomputing technology assessment, and provide a check for consistency of performance projections with assumed resources and fundamental physical constraints.

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