Eigenstate clustering around exceptional points

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Motivation

- **Theoretical Prediction**

non-Hermitian skin effect

PHYSICAL REVIEW LETTERS **121**, 086803 (2018)

Editors' Suggestion

Edge States and Topological Invariants of Non-Hermitian Systems

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Non-Hermitian Skin Effect

All eigenstates are localized around the edge in such a way that they become so close to each other that one can hardly distinguish them.

It was shown that not only topological edge states but also bulk states are exponentially localized around the boundaries in a nonreciprocal lattice with open edges. This extensively large density of eigenstates at edges implies that the standard bulk- boundary correspondence based on Bloch band topological invariants fails.

Motivation

Experimental realization

nature physics
ARTICLES
https://doi.org/10.1038/s41567-020-0836-6
Check for updates

Non-Hermitian bulk–boundary correspondence in quantum dynamics

Lei Xiao[1,5], Tianshu Deng[2,5], Kunkun Wang[1], Gaoyan Zhu[1], Zhong Wang[3] ✉, Wei Yi[2,4] ✉ and Peng Xue[1] ✉

PHYSICAL REVIEW RESEARCH **2**, 013280 (2020)

Photonic non-Hermitian skin effect and non-Bloch bulk-boundary correspondence

Xueyi Zhu,[1] Huaiqiang Wang,[2] Samit Kumar Gupta,[1] Haijun Zhang,[2,3,4]
Biye Xie,[1,*] Minghui Lu,[1,3,4,†] and Yanfeng Chen[1,3,4]

Motivation

Application

Science

REPORTS

Cite as: S. Weidemann *et al.*, *Science*
10.1126/science.aaz8727 (2020).

Topological funneling of light

Sebastian Weidemann[1*], Mark Kremer[1*], Tobias Helbig[2], Tobias Hofmann[2], Alexander Stegmaier[2], Martin Greiter[2], Ronny Thomale[2], Alexander Szameit[1†]

Questions:

What is the general condition of such closeness of eigenstates for a given non-Hermitian Hamiltonian?

Localization around any point in the lattice instead of edges. Can it be possible?

Can such a clustering be possible for a periodical system?

Answer

An N-th order exceptional point is a point singularity in the parameter space of an N-level system at which all eigenstates and their eigenvalues coalesce.

$$H = H_{EP} + \varepsilon H_1$$

H_{EP} has a Jordan block form

$$\psi_n = \psi^0 + \sqrt{\varepsilon}\ \psi_n^1 + \varepsilon\ \psi_n^2 + \ldots$$

Nature **548**, 192–196 (2017).

Around the exceptional point, the forms of the eigenstates become slightly different from each other. They can be hardly distinguishable from the experimental point of view.

Answer: Clustering

the systematic study of grouping the eigenstates in such a way that the eigenstates in the same group are more similar to each other than to those in other groups. In other words, the functional forms of any two eigenstates in two different groups should be different from each other as much as possible to have more meaningful grouping.

$$F_{nm} = \frac{| < \psi_n | \psi_m > |^2}{< \psi_n | \psi_n >< \psi_m | \psi_m >}$$

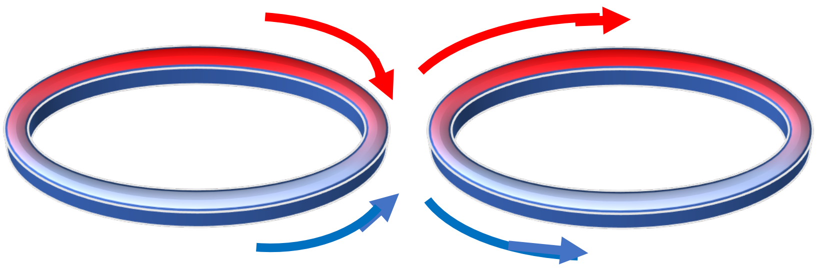
The signal of eigenstate clustering: the fidelities are piled up around 1.

Answer: Clustering

Here we use the k-means clustering algorithm

Suppose that there exists a set of M eigenstates $\{|\alpha >\}$, where the fidelities among them are either exactly or almost equal to zero. In this way, we construct M-dimensional data space. Then we find fidelities for the rest of eigenstates with respect to $\{|\alpha >\}$ to produce a set of data. Finally we apply the k-means clustering algorithm to make an unsupervised classification of the eigenstates.

illustration: 1D Lattice



Microring Resonator with clockwise and anticlockwise modes

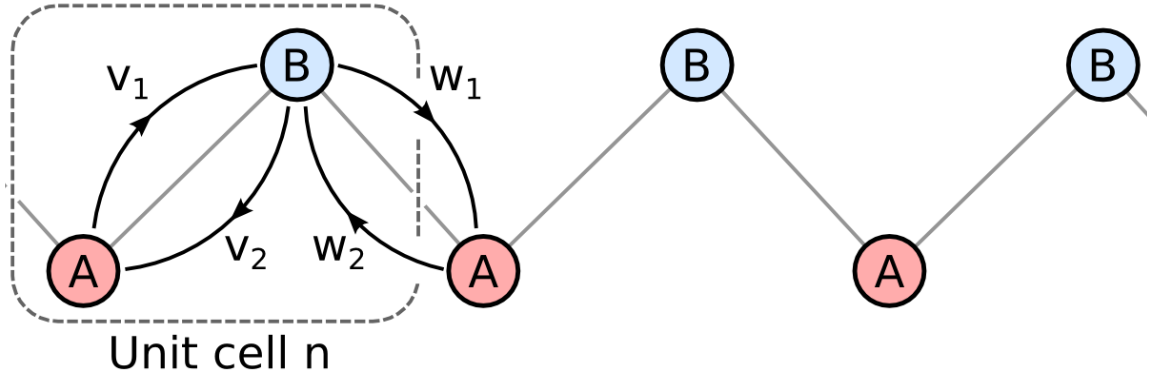
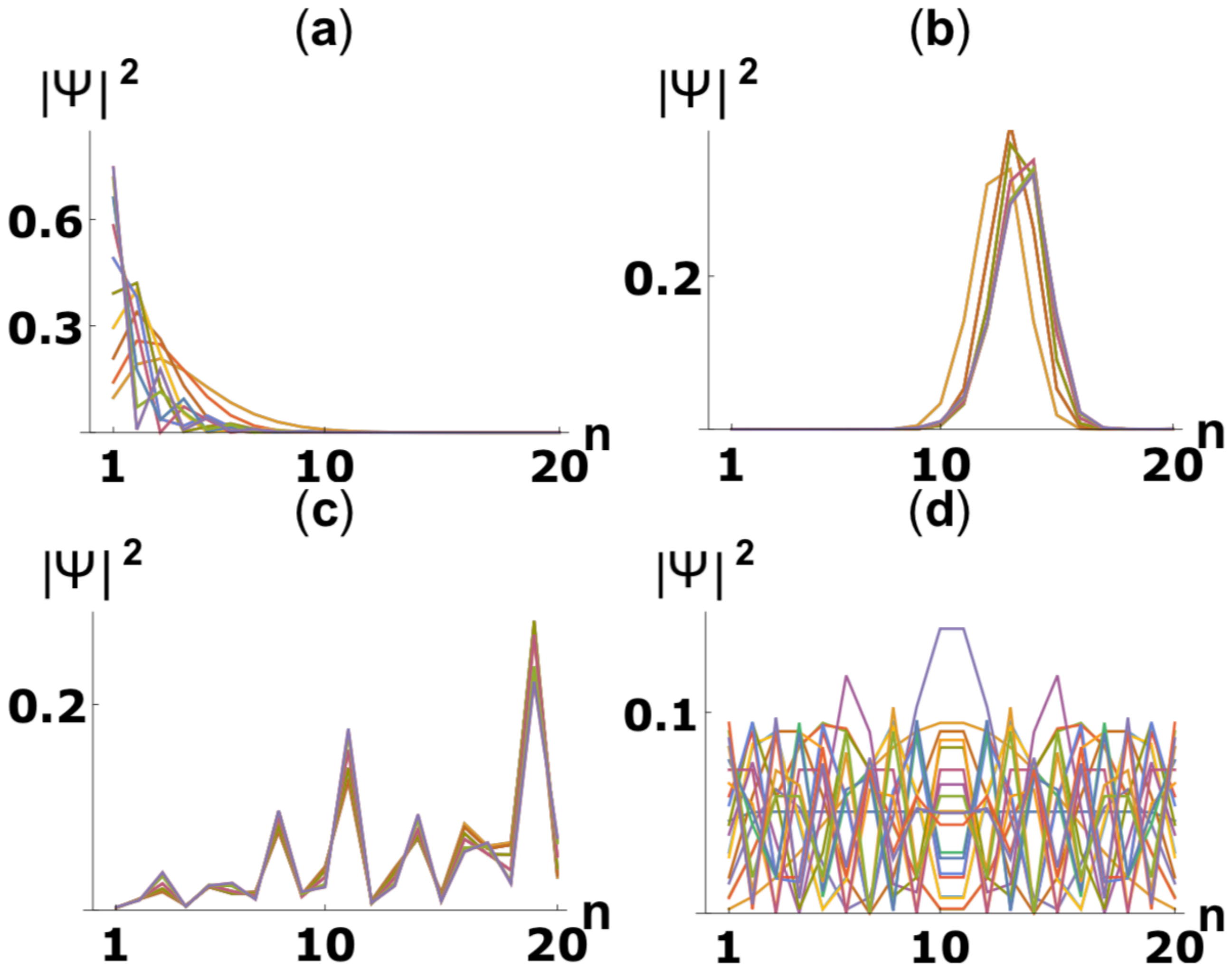


Illustration:



$$H\,\psi_n = J_n\,\psi_{n+1} + T_{n-1}\,\psi_{n-1}$$

$J_n = 0.1$ and $N = 20$ for all plots.

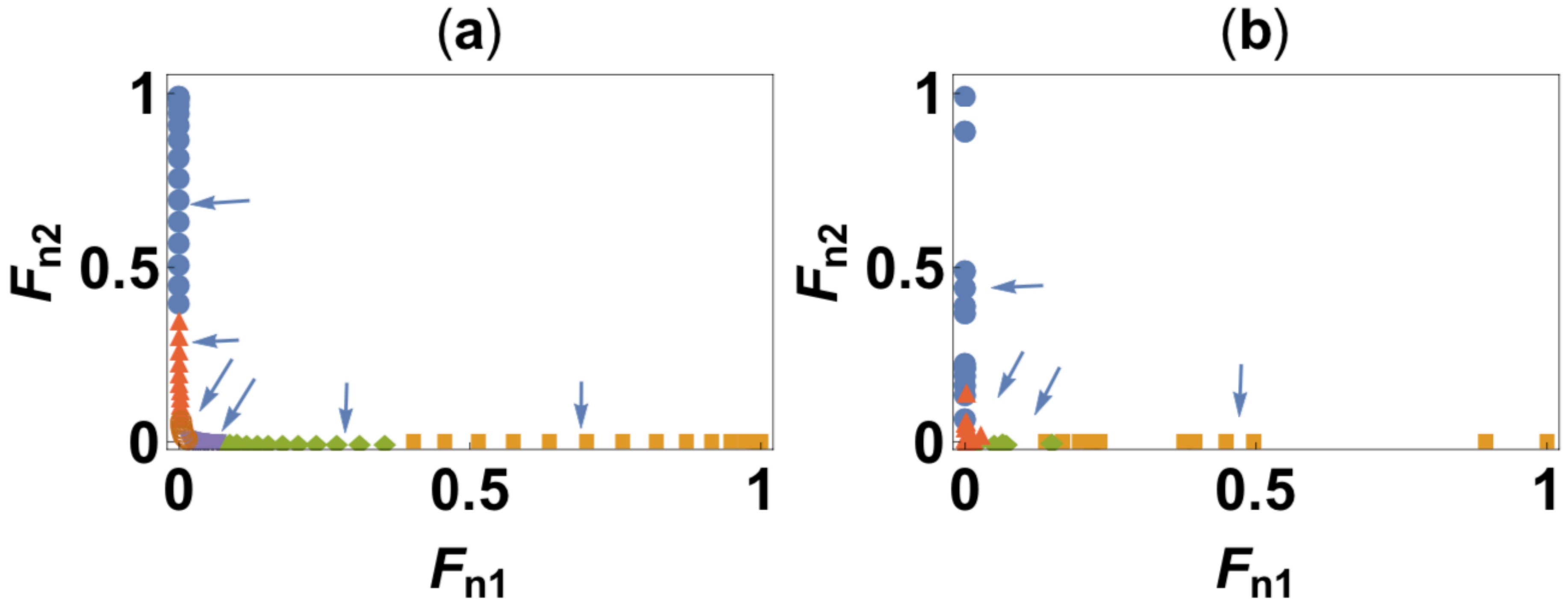
(a) $T_n = 0.05$, OBC

(b) $T_n = 0.1 + \sin 2(n/5)$, PBC

(c) $T_n = 0.1 + \sin 2(n/0.5)$, PBC

(d) $T_n = 0.1$, OBC

Clustering alghoritm



Conclusion:

- We have predicted eigenstate clustering of non-orthogonal eigenstates.
- We have discussed that non-Hermitian skin effect is a typical example of eigenstate clustering. We have found other examples.
- We have used a clustering algorithm of machine learning for the first time in the literature of non-Hermitian systems.

Future:

- Determination of topologically insulating states using machine learning.