Effect of Eccentric Dimple Potentials on Propagation, Squeezing and Localization of Matter Waves

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Abstract

We numerically investigate the time evolution of a matter wave in a harmonic trap decorated with an eccentric time independent and time dependent repulsive or attractive dimple potential. We also compute the time evolution of matter wave in one-dimensional optical lattice in the presence of repulsive dimple potential. One-dimensional Gross-Pitaevskii equation or nonlinear Schrodinger equation is used in order to get time evolution of the matter wave soliton. Dark and bright soliton can be generated by repulsive and attractive dimple potential respectively. Also, we theoretically demonstrate the posibility of the generation of nonclassical states of matter waves in a harmonic trap decorated with an eccentric dimple potential. We find that the squeezing can be controlled by the depth of the dimple potential if we neglect the atomic interactions. The variance of the system and Wigner function of the harmonic oscillator with a dimple potential are calculated analytically. Moreover, localization effect is observed in one-dimensional optical lattice in the presence of repulsive dimple potential.

Effective one dimensional Gross Pitaevski Equation:

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\partial_{zz}\psi+V(z,t)\psi+\frac{U_0N}{2\pi a_t^2}|\psi|^2\psi. \qquad (1)$$

$$U_0 = 4\pi\hbar^2 a_s/m \qquad a_t = \sqrt{\hbar/m\omega_t}$$

Time independent dimple potential models for matter wave

$$V(z)=\frac{1}{2}m\omega_z^2z^2+V_D\ \ (1-\frac{z^2}{a^2}). \qquad (2)$$

V_D is the strength (depth) of the dimple trap located at $z=0$.

Here a is the width of the dimple potential, ω_z is the trap frequency of the harmonic trap

$$V(z)\ =\ \frac{1}{2}m\omega_z^2z^2+V_D\ \ e^{-(\frac{(z-z_D)^2}{\sqrt{2}l_z})^2}, \qquad (3)$$

V_D is the strength (depth) of the dimple trap located at $z=z_D$.

Potential Models for matter wave

Time and position dependent dimple potential:

$$V(z,t) = \frac{1}{2}m\omega_z^2 z^2 - V_0 e^{-\left(\frac{z - z_D\cos(\Omega_D t)}{\sqrt{2}l_z}\right)^2}, \tag{4}$$

V_D is the strength (depth) of the dimple trap located at $z = 0$.

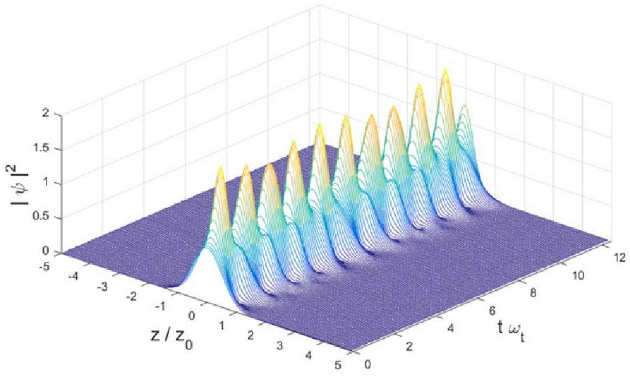
Time independent and repulsive dimple potential in an optical lattice:

$$V(z,t) = V_{OL}\cos(\vec{k}.\vec{z})^2 + V_D e^{-\left(\frac{z - z_0}{\sqrt{2}l_z}\right)^2}, \tag{5}$$

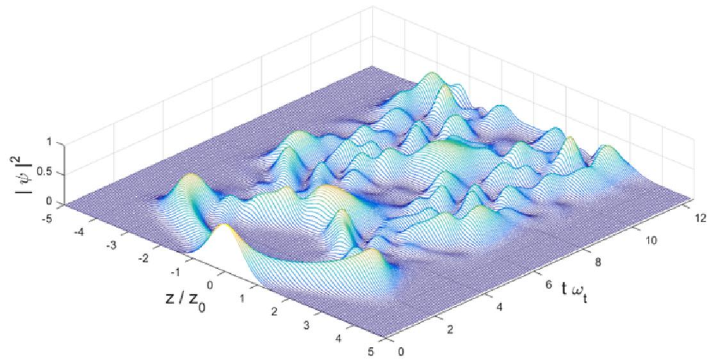
Results and Discussions

A) The propagation of bright and dark solitons:

$$z' = z/z_0,\ t' = \omega_t t\ ,\ z_0 = \sqrt{\hbar/m\omega_z}.$$



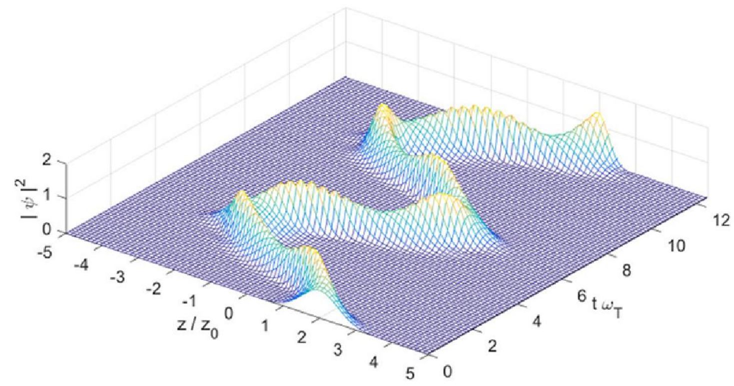
(a)



(b)

FIG. 1: (a) Time evolution of the matter wave. $V_D = -10\hbar\omega_t$ represents the attractive interactions. For dimensionless GPE equation $g = 0.02$ for the repulsive scattering length $a_s/a_z = 10^{-4}$ and $N = 10^3$ ^{23}Na atoms. (b) Time evolution of the matter wave. For dimensionless GPE equation $g = 0.02$ and $V_D = 10\hbar\omega_t$ represents the repulsive interactions.

A) The propagation of bright and dark solitons:



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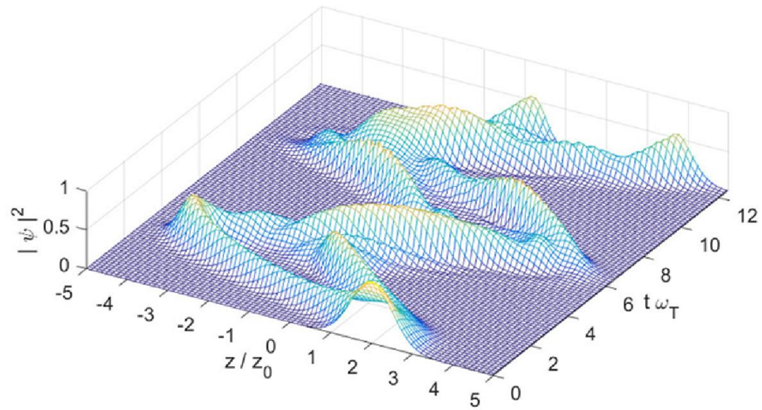


(b)

FIG. 2: (a) Time evolution of the matter wave. $V_D = -10\hbar\omega_t$ represents the attractive interactions. For dimensionless GPE equation $g = 0.02$ for the repulsive scattering length $a_s/a_z = 10^{-4}$ and $N = 10^3$ ^{23}Na atoms. The location of the dimple potential is $\omega_D = 3$ Hz. (b) Time evolution of the matter wave. For dimensionless GPE equation $g = 0.02$. $V_D = 10\hbar\omega_t$ represents the repulsive interactions.

A) The propagation of bright and dark solitons:

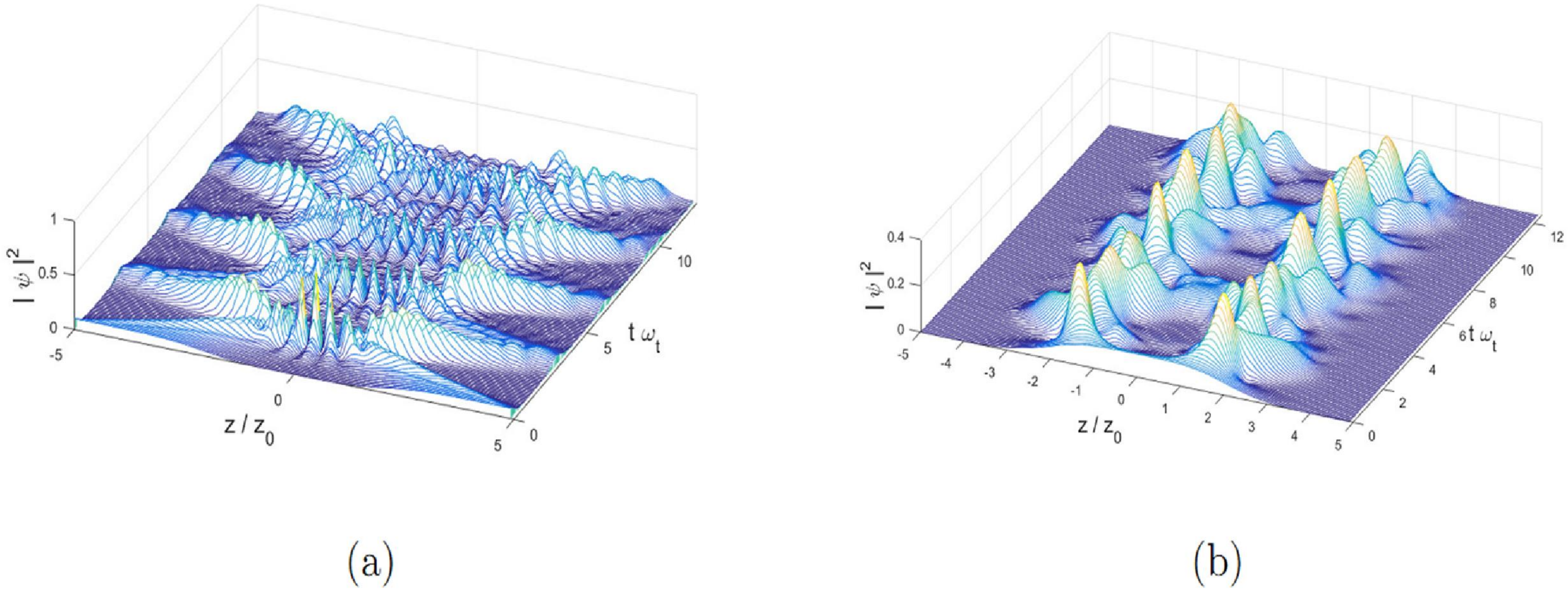


FIG. 4: (a) We take initial wave function as ($\psi = \sqrt{(\kappa/2)}tanh(\kappa x)$) for the time evolution of the matter wave. For dimensionless GPE equation $g = 0.02.V_D = -10\hbar\omega_t$ represents the attractive interactions and bright soliton solution. (b) ($\psi = \sqrt{(\gamma/2)}sech(\gamma x)$) is taken as initial wave function for the propagation of the matter wave. For dimensionless GPE equation $g = 0.02.$ $V_D = 10\hbar\omega_t$ represents the repulsive interactions and repulsive dimple respectively.

B) Squeezing of matter wave:

If we neglect the atomic interactions, we can use the time independent Scrödinger Equation for external potential given by Eq.(2). In Eq.(2) for $V_D = 0$, the eigenstates and eigenvalues of the Hamiltonian is $\phi = (\alpha/(\sqrt{\pi}2^n n!))^{1/2} H_n(\alpha z) e^{-(1/2)\alpha^2 z^2)}$ and $E_n = \hbar\omega_z(n + 1/2)$ respectively. Here $\alpha = (m\omega_z/\hbar)^{1/2}$, $H_n(\alpha z)$ is n^{th} Hermite Polynomial and ω_z is the trap angular frequency. For $|\leq a$ the eigenstates and eigenvalues of the Hamiltonian are $\phi = c_1 D_\lambda(z) + c_2 D_\lambda(-z)$ and $\lambda = (E + V_D)/(\hbar\omega_D) - 1/2$ respectively. Here $D_\lambda(z)$ is parabolic cylinder function and $\omega_D = \sqrt{\omega_z^2 + 2V_D/(ma^2)}$ [33]. Solving the Heisenberg equa-

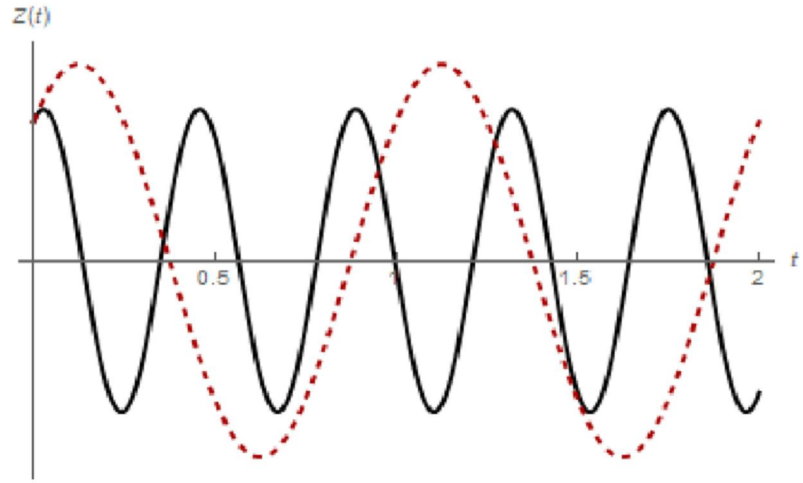
$$\left.\begin{aligned} \frac{dz}{dt} &= \frac{p}{m}, \\ \frac{dp}{dt} &= -m\omega_D^2 z. \end{aligned}\right\} \quad \begin{aligned} z(t) &= z(0)\cos(\omega_D t) + \frac{p(0)}{m\omega_D}\sin\omega_D t, \\ p(t) &= p(0)\cos(\omega_D t) - m\omega_D z(0)\sin\omega_D t. \end{aligned}$$

B) Squeezing of matter wave:

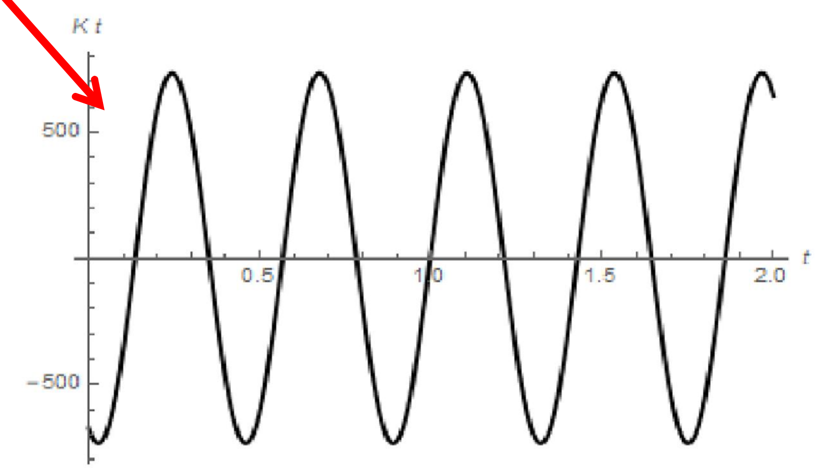
Variation:

$$\kappa_z(t) = \frac{< z^2(t) > - < z(t) >}{z^2(0)}.$$

$$< z^2(0) >= \hbar/(2m\omega_D)$$



(a)



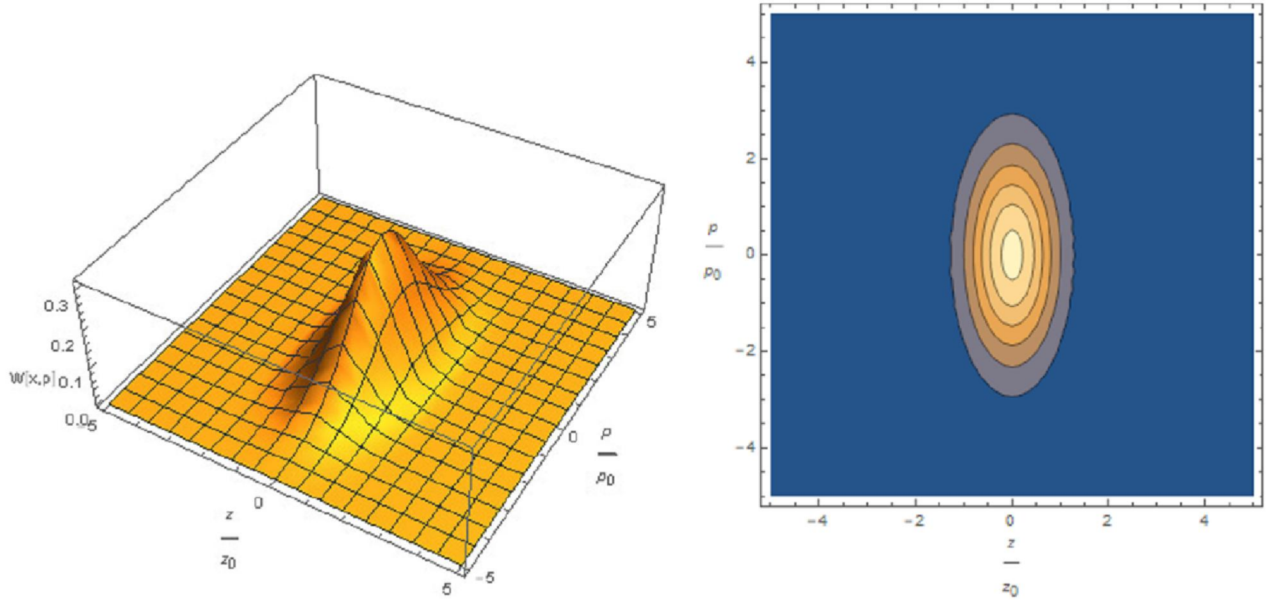
(b)

FIG. 5: (a) $z(t)$ as a function of time. $V_D = 0$ solid $V_D = 50\hbar\omega_t$ (dashed) (b) Time dependence of the variation written in Eq. (10).

B) Squeezing of matter wave:

Wigner Function:

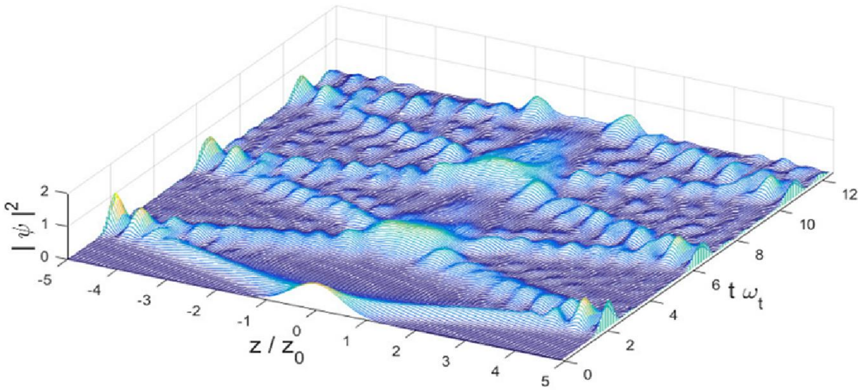
$$W(z,p) = \frac{(-1)^n}{\pi\hbar} \exp\left[\frac{-2n}{\hbar\omega_D}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega_D^2 z^2\right)\right] \times$$
$$\times L_n\left[\frac{4}{\hbar\omega_D}\left(\frac{p^2}{2m} + \frac{1}{2}m\omega_D^2 z^2\right)\right].$$



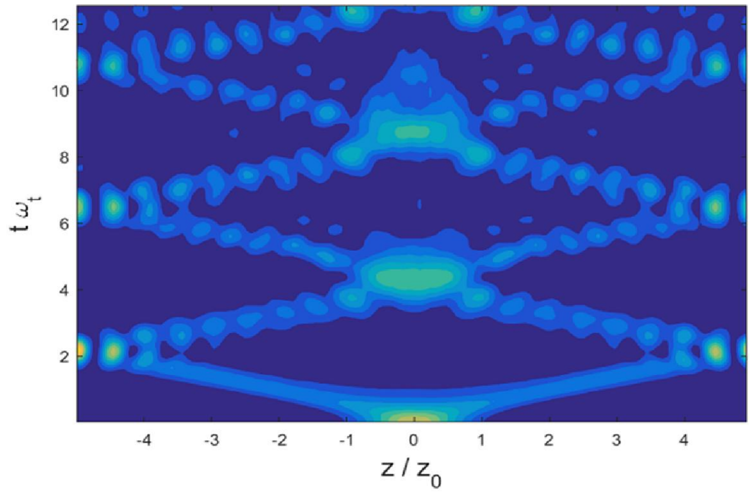
(a) (b)

FIG. 6: (a) Dependence of Wigner Function $W(z,p)$ on a momentum and space for $V_D = 50\hbar\omega_t$ (b)Contour plot of Wigner Function.

C) Localization of Matter wave:



(a)



(b)

FIG. 8: (a) We assume a Gaussian wave function which is the ground state at initial time t_0. The propagation of the matter wave in an optical lattice is shown. For dimensionless GPE equation $g = 0.02$ and $V_D = 10\hbar\omega_t$ represent the repulsive interactions and repulsive dimple respectively. (b) Contour graph of the time evolution of the matter wave. All parameters are the same as Fig. (8(a)).

Conclusion:

In conclusion, we have investigated numerically the matter wave by solving GPE that describes the interacting matter wave in the quasi 1D harmonic potential with eccentric dimple potential. We have studied time propagation of the matter wave in a harmonic trap with dimple potential whose location is time dependent. This procedure can be used for the generating the accelerated matter waves and squeezed states of the matter wave.

Squeezing of the matter wave can be achieved by applying time independent dimple potential. Single-mode squeezed states may be generated by eccentric dimple potentials. The eccentric dimple potential in the harmonic trap can lead to the squeezing of matter wave. The eccentric dimple potential can also be used for coherent control of optical information with the matter wave dynamics by manipulating light with the matter wave. Light pulse storage and revival process can be achieved in two seperate BEC thanks to eccentric dimple potential in a harmonic trap. Therefore two mode nonclassical entangled state may be prepared by eccentric dimple potentials.

We further showed that a dimple potential is useful to provide localization effect in the one dimensional optical lattice. We have examined the time propagation of matter wave in an optical lattice with the repulsive dimple potential. As a result, localization effect can be observed in the existence of a repulsive dimple potential in an optical lattice.

Acknowledgement

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