Entanglement Analysis of 2-D Quantum Walk with Entangled Coins

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1-D Quantum Walk vs Classical Walks

A random walk is the process that objects move randomly away from where they started. The simplest example of a random walk is the classical motion of a particle on a line, the direction of which is determined by a non-biased coin. [1] Quantum walk on a line is the quantum analog of classical random walk on a line. In contrast to the classical random walk, the direction of motion in a quantum walk depends on the state of a quantum coin. One of the simplest realizations of a quantum coin is the Hadamard transform that maps the coin-basis states $|0\rangle$ and $|1\rangle$ to two superposition states with equal weight basis states $|0\rangle$ and $|1\rangle$. [2]

For a quantum walk on a line, the walker's position n should be a vector in a Hilbert space $\mathcal{H_P}$ of infinite dimension, the computational basis of which is $|n\rangle : n \in \mathbb{Z}$ and The state of coin should be a basis vector ($|0\rangle$ or $|1\rangle$) in a the two dimensional Hilbert space $\mathcal{H_C}$. Therefore the total Hilbert space of the system will be $\mathcal{H} = \mathcal{H_P} \otimes \mathcal{H_C}$.

The evolution of the walk depends on the quantum coin. If one obtains $|0\rangle$ after tossing the quantum coin and the walker is described by $|n\rangle$, then in the next step it will be described by $|n+1\rangle$. If it is $|1\rangle$ it will be described by $|n-1\rangle$.

The shift from $|n\rangle$ to $|n+1\rangle$ or $|n-1\rangle$ must be described by unitary operator. It is called the shift operator

$$S = |0\rangle\langle 0| \otimes \sum_{n=-\infty}^{\infty} |n+1\rangle\langle n| + |1\rangle\langle 1| \otimes \sum_{n=-\infty}^{\infty} |n-1\rangle\langle n|$$

$$S|0\rangle|n=0\rangle = |0\rangle|n+1\rangle$$

$$S|1\rangle|n=0\rangle = |1\rangle|n-1\rangle$$

One of the coin operators for one-dimensional quantum walk is the Hadamard Transform

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

Initial State $|\Psi(0)\rangle = |0\rangle \otimes |n=0\rangle$

$$(H \otimes I)|\Psi(0)\rangle = \frac{(|0\rangle+|1\rangle)}{\sqrt{2}} \otimes |n=0\rangle$$

$$S(H \otimes I)|\Psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |n=+1\rangle + |1\rangle \otimes |n=-1\rangle)$$

The result is superposition of particle's state $|n=1\rangle$ and $|n=-1\rangle$. If one measures the final state using the computational basis of $\mathcal{H_P}$, one will have equal chance of finding the walker at position $n = 1$ and $n = -1$. The evolution operator is $U = S(H \otimes I)$. At time t, the state of walker is given by $|\Psi(t)\rangle = U^t|\Psi(0)\rangle$.

2-D Quantum Walk vs Entangled Coins Walk

For 2-D quantum walk with initially entangled coins, the state of coins is one of the Bell States $\left(|\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$ which are maximally entangled pure bipartite states. The coin operator for 2-D is defined as the tensor product of two single coin operators: $C = H \otimes H$[2]. The shift operator is a version of quantum walk on a line;

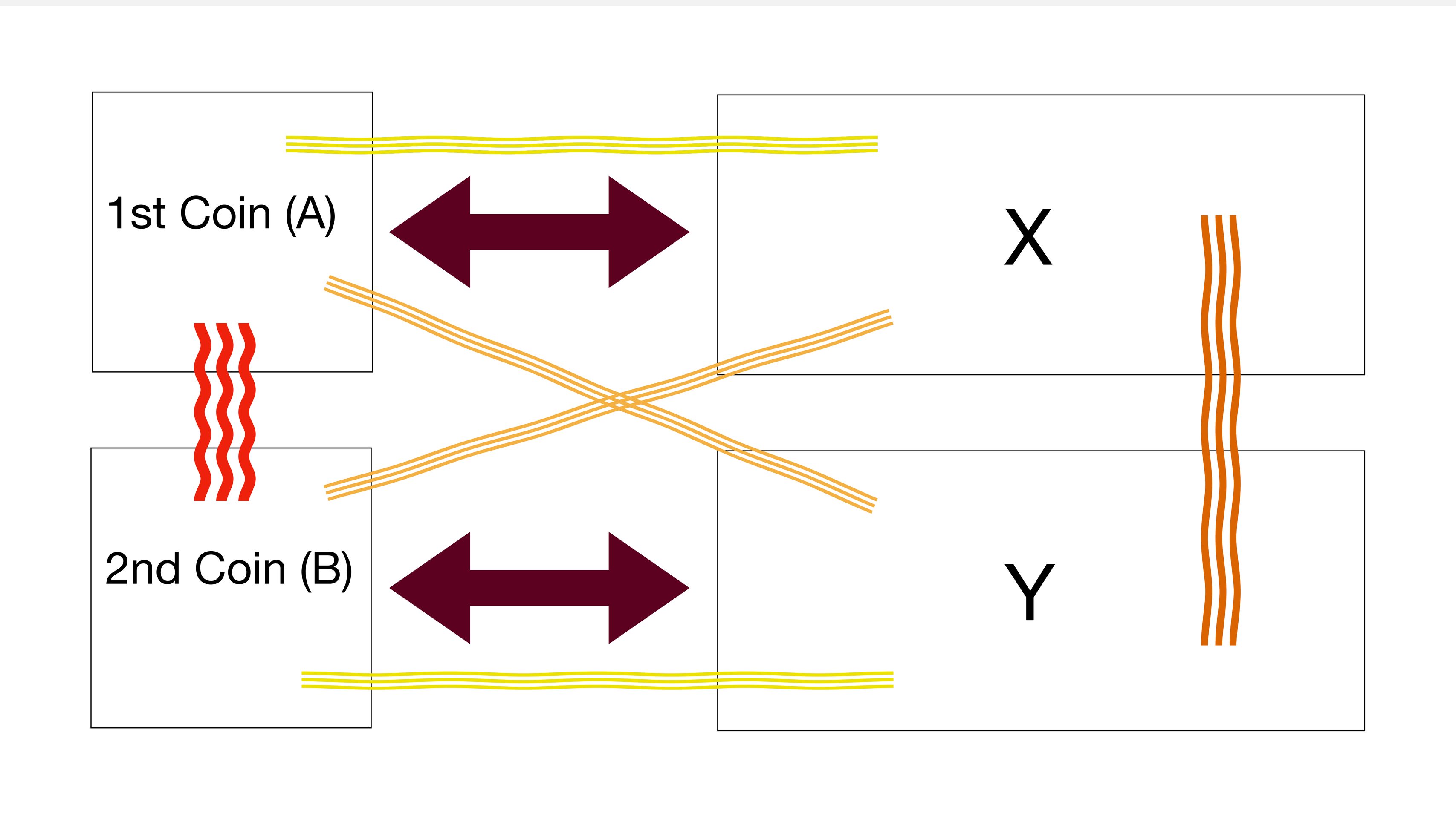
$$S = |00\rangle\langle 00| \otimes \sum_{x,y=-\infty}^{\infty} |x+1,y+1\rangle\langle x,y| + |01\rangle\langle 01| \otimes \sum_{x,y=-\infty}^{\infty} |x-1,y+1\rangle\langle x,y|$$

$$+ |10\rangle\langle 10| \otimes \sum_{x,y=-\infty}^{\infty} |x+1,y-1\rangle\langle x,y| + |11\rangle\langle 11| \otimes \sum_{x,y=-\infty}^{\infty} |x-1,y-1\rangle\langle x,y|$$

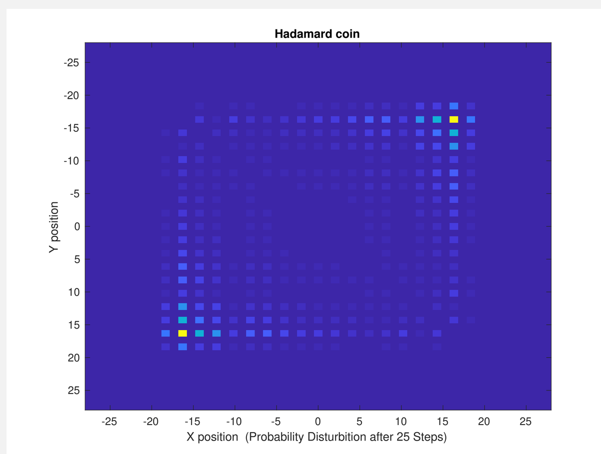
We choose initial state as; $|\Psi\rangle_0 = |\phi^+\rangle \otimes |x=0\rangle\,|y=0\rangle$, The evolution operator $U = S(C \otimes I \otimes I)$. Therefore; $|\Psi(t)\rangle = U^t|\Psi(0,0)\rangle$.

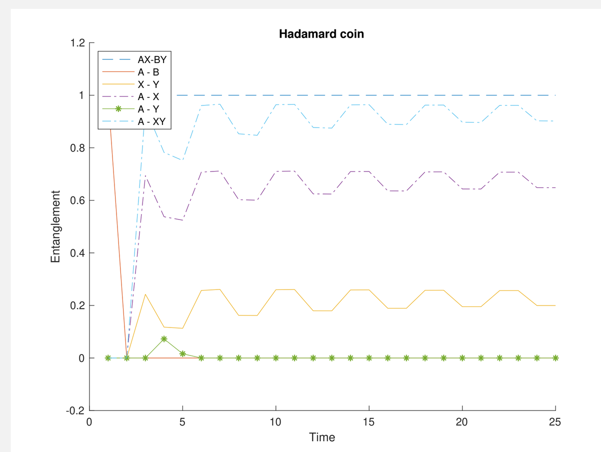
Entanglement Transfer

We present an analysis for a quantum walk with two entangled coins on 2 dimensions with different coins. We studied 2-D quantum walk analytically and numerically to investigate the transform of the entanglement in initial coins state to spatial degreees of freedom. We present our results that plots the probability distributions of a 2-D quantum walk whose initial coin states are chosen as one of the Bell states.

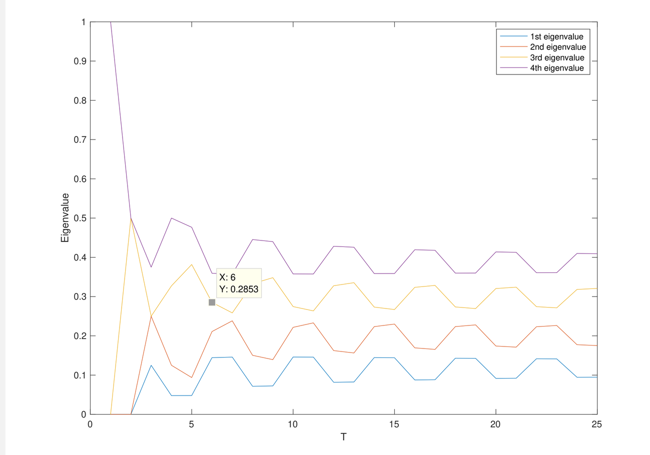


• Diagram of Transfer

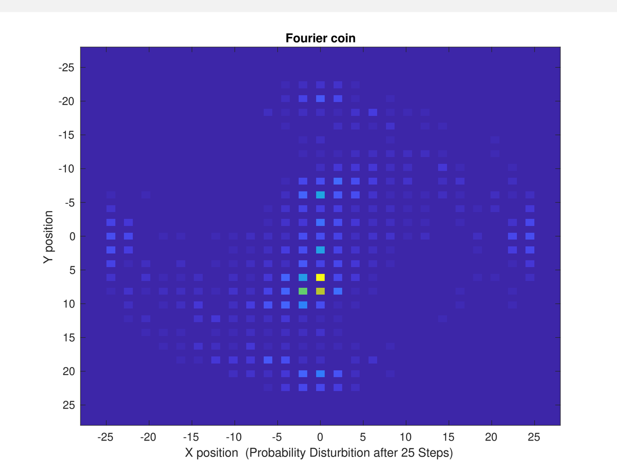


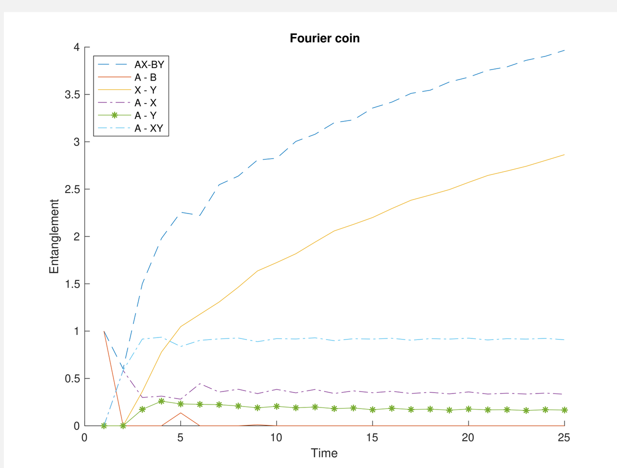


HADAMARD COIN

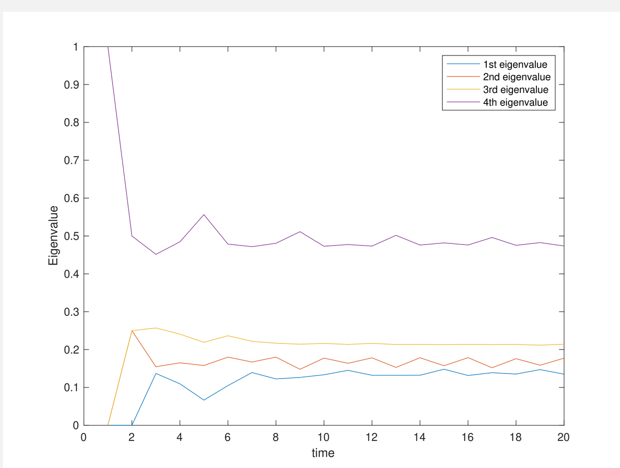


$$H_C = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

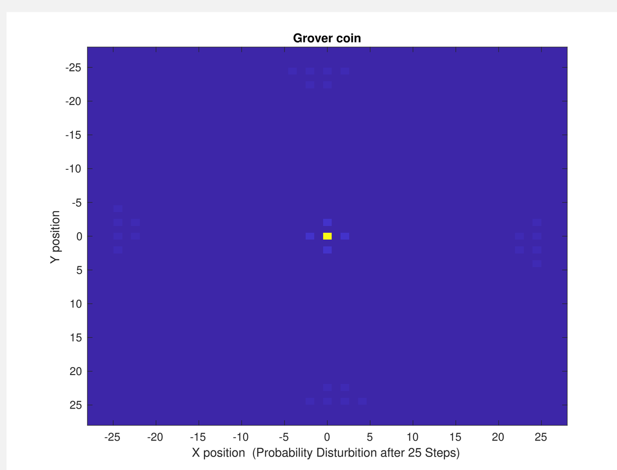


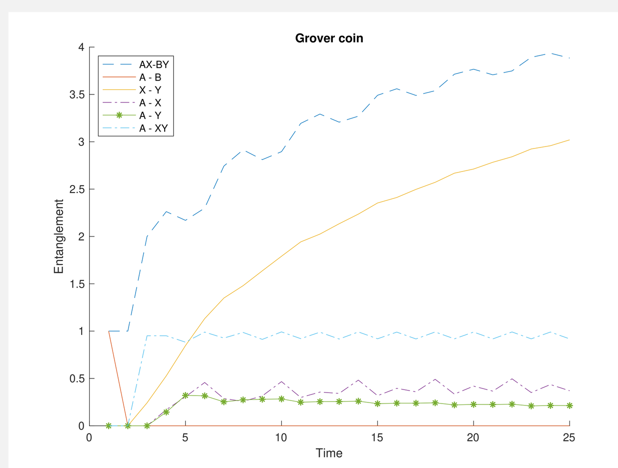


FOURIER COIN

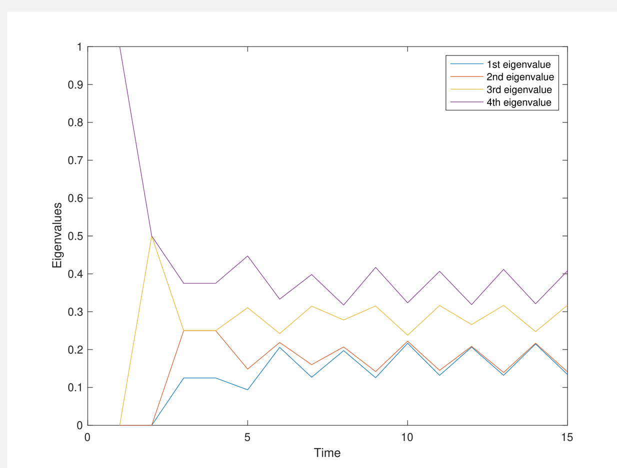


$$F_C = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & i \end{pmatrix}$$





GROVER COIN



$$G_C = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

[1] Portugal, R.: Quantum Walks and Search Algorithms, Springer (2013)

[2]Venegas-Andraca, S. E. (2012). Quantum walks: A comprehensive review. Quantum Information Processing