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Small-Scale Artificial Neural Networks for Statistical Modeling and Controling Quantum Systems

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Main Concept: Using ANNs based on non-linear artificial neurons with threshold properties for statistical modeling of quantum systems.

Keywords: Artificial Neural Networks, Quantum Dynamical Systems, Feedback Control

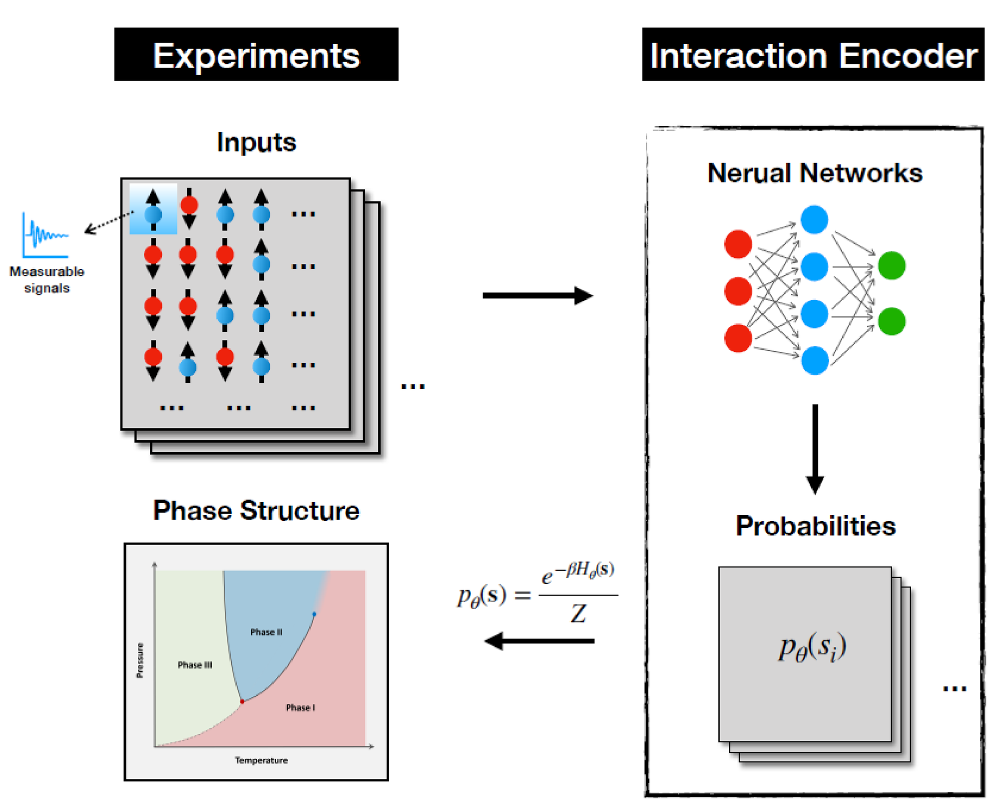
ABSTRACT

Very recently a new statistical physics approach based on Artificial Neural Networks (ANNs) has been proposed by Wang, Jiang, and Zhou in 2020 for the particular case of a cyclic Ising system. A small-scale ANN is well-trained with the data collected from the '*Ab Initio*' experiment or numerical simulations for mimicking the microscopic statistical states of a quantum system. Then it can be spontaneously extrapolated to evaluate macroscopic states of the quantum system, its phase structures, and thermodynamic characteristics.

We study alternative small-scale ANN approaches (Masked Autoencoder for Distribution Estimation, Hodgkin-Huxley neural networks) to describe the statistical micro- and macro-states of different quantum systems (qubits, Ising spin systems, memristors), and their possible applications to control over the quantum system dynamics.

Additionally, we discuss the emulation of quantum algorithms with a small population of classical non-linear artificial neural elements (like Hodgkin-Huxley neurons) and their possible application to control other quantum and classical systems.

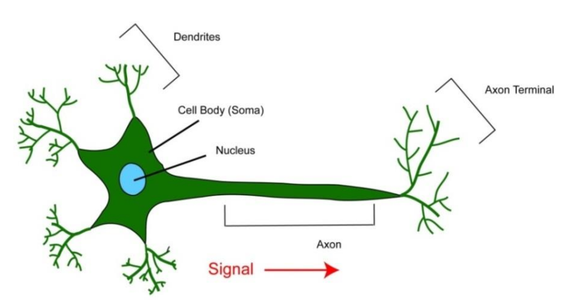
Paradigm:



Wang, L., Jiang, Y., Zhou, K. 2020. Neural network statistical mechanics. arXiv:2007.01037 [physics.comp-ph].

The inputs are collected from experiments or the first-principle calculations.
The inputs are magnetic domain fed to the encoder.
The interactions are encoded by the neural networks which produces the probability distributions of the microscopic states.
The configurations are sampled at different external parameters with the well-trained networks, which predicts the phase structure.

Basic Tool: Hodgkin - Huxley Neurons



$v(t)$ stands for the **membrane potential**, $m(t)$, $n(t)$, $h(t)$ are the **membrane gate variables**, and the control signal is represented by the sum $I(t)$ of **external and synaptic currents** entering the cell.

$$C_M \cdot \frac{dv}{dt} = -g_{Na} m^3 h \cdot (v - E_{Na}) - g_K n^4 \cdot (v - E_K) - g_{Cl} \cdot (v - E_{Cl}) + I \;;$$

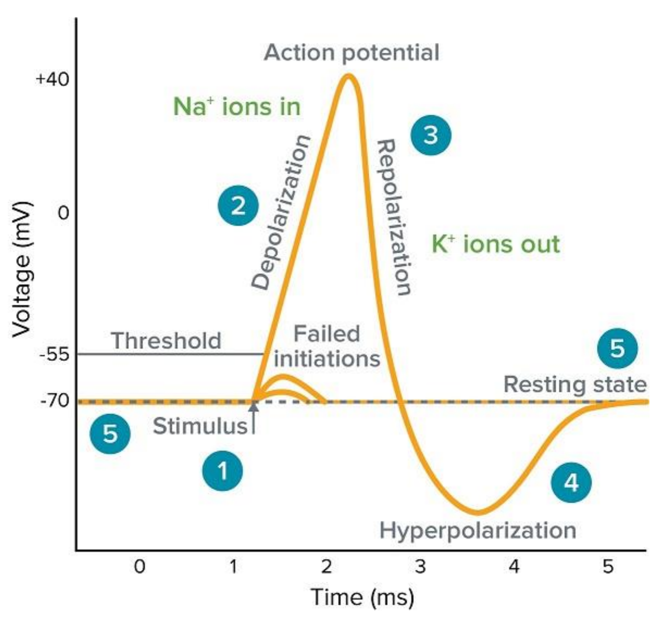
$$\frac{dm}{dt} = \alpha_m(v)\cdot(1-m) - \beta_m(v)\cdot m \;;$$

$$\frac{dn}{dt} = \alpha_n(v)\cdot(1-n) - \beta_n(v)\cdot n \;;$$

$$\frac{dh}{dt} = \alpha_h(v)\cdot(1-h) - \beta_h(v)\cdot h \;.$$

$v(t)$ = Membrane potential
$m(t)$, $n(t)$, $h(t)$ = Ion channels variables
I = External and synaptic currents entering the cell
E_{Na} = Equilibrium potential at which the net flow of Na ions is zero
E_K = Equilibrium potential at which the net flow of K ions is zero
E_{Cl} = Equilibrium potential at which leakage is zero
C_M = Membrane capacitance
g_{Na} = Sodium channel conductivity
g_K = Potassium channel conductivity
g_{Cl} = Leakage channel conductivity
$\alpha, \beta_{m,n,h}(v)$ = Suitable rate functions

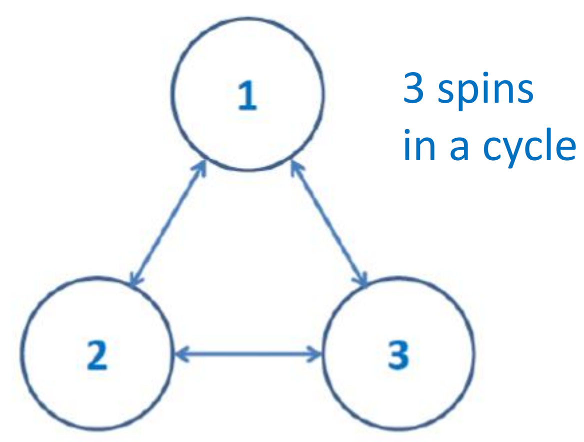
Existence of ***thresholds***: If the current overcomes a minimum threshold level, the HH neuron produces a spike; for the current stimulus above a certain greater level, the outcome is a spike train, etc.



Hodgkin, A. L., Huxley, A. F. 1952. Currents carried by sodium and potassium ions through the membrane of the giant axon of Loligo. The Journal of Physiology, 116 (4), 449-472.

Example 1: Ising Ferromagnetic Model

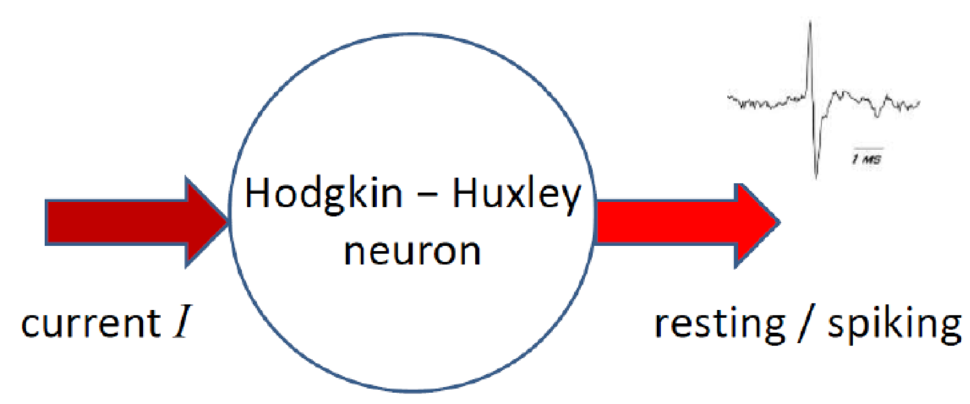
$$H_\theta(\mathbf{s}) = -T \log p_\theta(\mathbf{s})$$



$$p(s_1) \propto e^{-2s_1-1} + 2e + e^{2s_1-1}$$

$$p(s_2|s_1) \propto \frac{e^{-s_1 s_2}(e^{s_1+s_2} + e^{-s_1-s_2})}{e^{-2s_1-1} + 2e + e^{2s_1-1}}$$

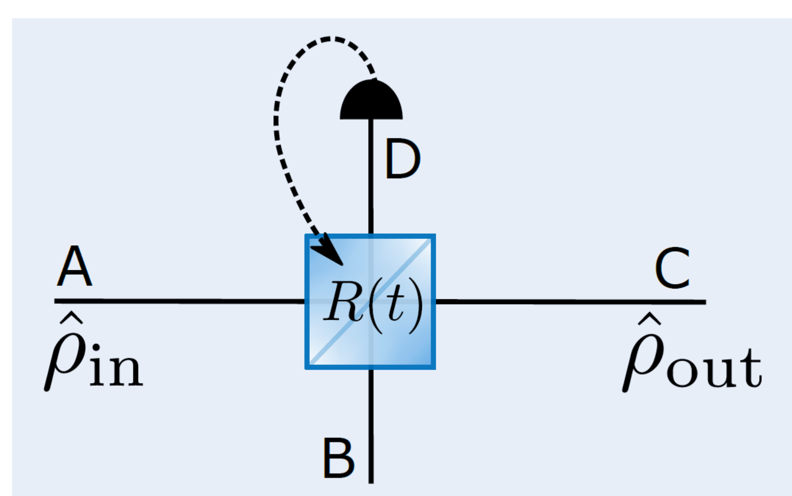
$$p(s_3|s_1, s_2) \propto \frac{e^{-s_3(s_1+s_2)}}{e^{s_1+s_2} + e^{-s_1-s_2}}$$



The probabilities to get a resting / spiking outcomes for the HH neuron exactly reflect the probabilities of the corresponding macrostates.

$$I = \frac{(s_1 s_2 + s_2 s_3 + s_3 s_1) + 1}{4} \cdot I_{\text{threshold}}$$

Example 2: Photonic Quantum Memristor (PQM)



Spagnolo, M., Morris, J., Piacentini, S., Antesberger, M., Massa, F., Ceccarelli, E., Crespi, A., R. Osellame, R., Walther, P. 2021. Experimental quantum memristor, arXiv:2105.04867v2 [quant-ph], 2021.

PQM contains the input part (the mode A) and two output parts (the modes C and D). The reflectivity $R(t)$ can be updated based on the measurement made at the mode D.

$$|\psi_{\text{in}}(t)\rangle = \alpha(t)\cdot|0\rangle_A + \beta(t)\cdot|1\rangle_A$$

$$|\alpha(t)|^2 + |\beta(t)|^2 = 1$$

The **purity** $P(t)$ indicates how close the outcome C is to a pure state:

$$P(t) = \text{Tr}\left(\rho^2_{\text{out}}(t)\right) = 1 - 2|\beta(t)|^4 R(t)(1 - R(t))$$

$$\langle n_{\text{out}}(t)\rangle = [1 - R(t)]\langle n_{\text{in}}(t)\rangle$$

$$\frac{dR(t)}{dt} = \langle n_{\text{in}}(t)\rangle - \frac{1}{2}\langle n\rangle_{\max}$$

$$\langle n_{\text{in}}(t)\rangle = |\beta(t)|^2 = u(t)$$

$$G(t) = \frac{1}{2}(R(t) - R_*)^2$$

$$u(t) = -\Gamma \frac{\partial}{\partial u}\left(\frac{dG(t)}{dt}\right) + u_0$$

$$\Gamma = \text{const} \geq 0 \;;\; u_0 = \text{const}$$

$$u = -\Gamma(R - R_*) + u_0$$

$$u_0 = \frac{1}{2}\langle u\rangle_{\max}$$

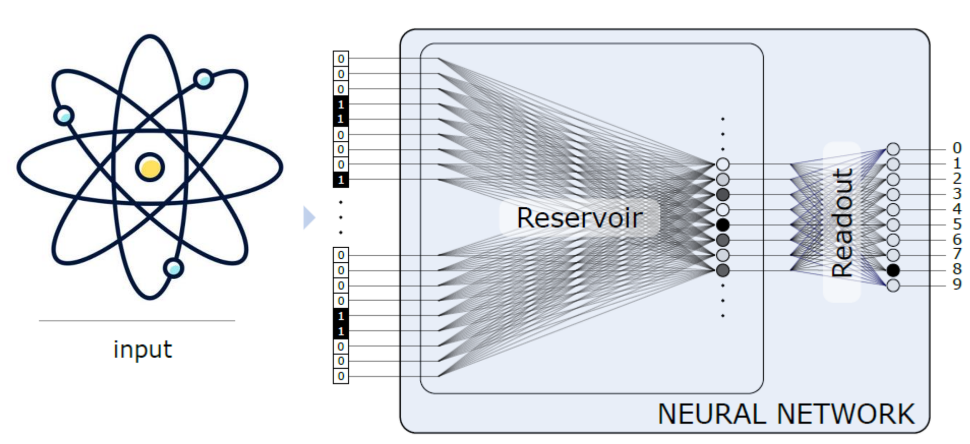
Speed Gradient

Fradkov, A. L. 2007. Cybernetical physics: From control of chaos to quantum control" Springer..

$$I(t) = \frac{1}{2}\left[1 - (-1)^{\theta(1-R(t)+\Delta) - \theta(1-R(t)-\Delta)}\right] \cdot I_t$$

The spiking component of the HH neuron reflects the coefficient $1 - R(t)$.

Perpectives: Quantum Reservoir Computing



The output pattern must be interpreted by a trainable readout network. Usage of a non-linear element with threshold properties can drastically decrease its complexity and the time of training, and improve its reliability.

Main Reference:
Borisenok, S. 2021. Statistical physics approach to small scale artificial neural networks. Proc. of 5-th International Online Conference on Mathematics "An Istanbul Meeting for World Mathematicians", Dec. 1-3, 2021, pp. 572-579.