Single-Atom Quantum Heat Engine in Asymmetric Harmonic Oscillator Trap by Optomechanical-like model

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ABSTRACT

Quantum heat engines (QHE'S) are thermal machines harnessing energy resources using quantum working substances [1, 2]. While the possibility of treating a three-level maser, a quantum device that amplifies microwave radiation, as a conventional heat engine was recognized several decades ago [3], systematic studies of QHE's have been attracted much attention quite recently [4]. A pioneering experiment was demonstrated as a piston-type (Otto) engine using a single trapped ion [5]. Due to the relatively small energy gaps of the trapped ion at the involved temperatures, genuine, profound quantum effects in the single-atom piston are assumed to be non-existent. On the contrary, here we question this conclusion by examining the quantum mechanical model of the interactions between the motional degrees of freedom of the atom.

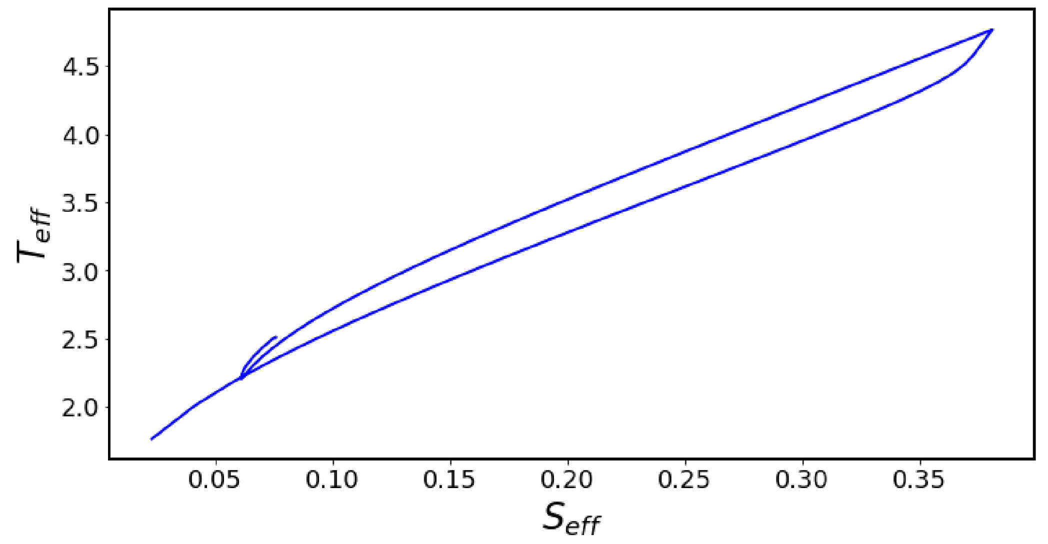
WORK & EFFICIENCY

The introduced 4-stage description can be considered as an Otto engine. The entropy of the mode a is calculated by [6]

$$S_a = (1+\langle \hat{n}_r \rangle)\ln(1+\langle \hat{n}_r \rangle) - \langle \hat{n}_r \rangle \ln(\langle \hat{n}_r \rangle).$$

Also, the effective temperature can be considered as

$$T_{\rm eff} = \omega_{\rm eff}/\ln(1+1/\langle \hat{n}_r \rangle).$$



The temperature-entropy the $T-S$ diagram, which closely resembles that of an Otto engine. The potential work output can be obtained by calculating the area of the T-S diagram.

The parameters used in the plots are taken to be $\beta = 400$ MHz, $\omega_r = 10$ GHz, $\omega_z = 500$ MHz, the mean number of excitations $\bar{n}_a = \bar{n}_b = 0.01$, $\bar{n}_h = 0.325$, and also system bath coupling $\kappa_a = \kappa_h = 2$ GHz, $\kappa_b = 50$ MHz.

We approximate the work from T-S diagram and the net work is estimated by $W_r \approx 0.10\hbar\omega_r \approx 1.12 \times 10^{-25}$joules. The Power can be calculated by dividing work by the cycle time as $P \approx 1.12 \times 10^{-16}$Watts. Also, the efficiency is $\eta \approx 0.093 = 9.3\%$.

MODEL SYSTEM

In the limit of $gz \ll 1$, the asymmetric trap potential

$$\mathrm{U}(x,y,z) = \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2)$$

can be approximated by

$$V(r,z) = \frac{m}{2}\omega_r^2 r^2(1+gz) + \frac{m}{2}\omega_z^2 z^2$$

We neglected the small difference between ω_x and ω_y and replace them with the mean radial trap frequency, $\omega_r := (\omega_x + \omega_y)/2$ [24].

We apply the harmonic-oscillator quantization to write the Hamiltonian in terms of the vibrational phonon operators, (we take $\hbar = 1$)

$$H_{CM} = n_r\omega_r + n_z\omega_z - \beta\left(a+a^\dagger\right)^2(\mathrm{b}+b^\dagger)$$

Where $\beta = g\omega_r\sqrt{1/2m\omega_z}$, is the coupling coefficient of r and z degrees of freedom; and n_r and n_z are the phonon number operators in r and z directions, respectively. The Hamiltonian (5) can be compared.

QUANTUM HEAT ENGINE

To identify the Otto engine cycle in our model, we define an effective frequency of the working fluid (mode a) [6]. Accordingly, the effective frequency of the motion in mode a will be $\omega_{eff} = \sqrt{\omega_x^2 + 2\omega_x\beta q_y}$, where $q_y = (b + b^\dagger)$.

The Otto engine is described with four steps [7]. (A→B) Hot isochore: the radial trapping frequency (ω_r) is kept constant at ω_h while contacting with hot bath at temperature T_h. The termalization will be done after a time of τ_h . (B→C) Adiabatic expansion: The atom evolves in the trapping potential while isolated from the hot bath. After a time τ_z, the radial frequency will be changed from ω_r to ω_c and a work will be done by considering the displacement in axial direction. (C→D) Cold isochore: the radial frequency remain in ω_c and the cold bath with temperature to T_c is contacted with the system for a time to τ_c. (D→A) Adiabatic compression: the radial frequency returns to initial frequency in another time to τ_c in last step.

Coupling coefficient

The Figure shows how the coupling coefficient affects on extracted work of the engine that is increasing by increasing the to β. By considering the definition of the coupling coefficient $\beta = g\omega_r\sqrt{1/2m\omega_z}$, it is clear that β depends on g and ω_r linearly, and axial frequency's dependency is $\beta \propto 1/\sqrt{\omega_z}$.

In an experimental model, the atom can be considered trapped in an asymmetric Paul trap, based on a single-atom heat engine experiment [5]. The coupling constant of potential g can be considered as $g = 4\,tan\theta/r_0$ where r_0 is the radius of the trap at z = 0 and θ is the trap angle.

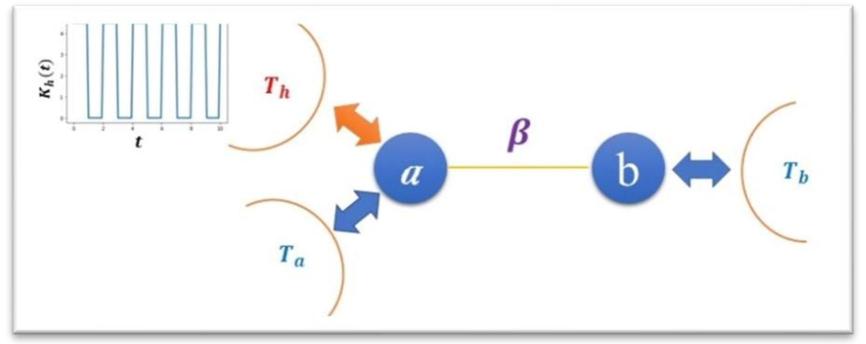
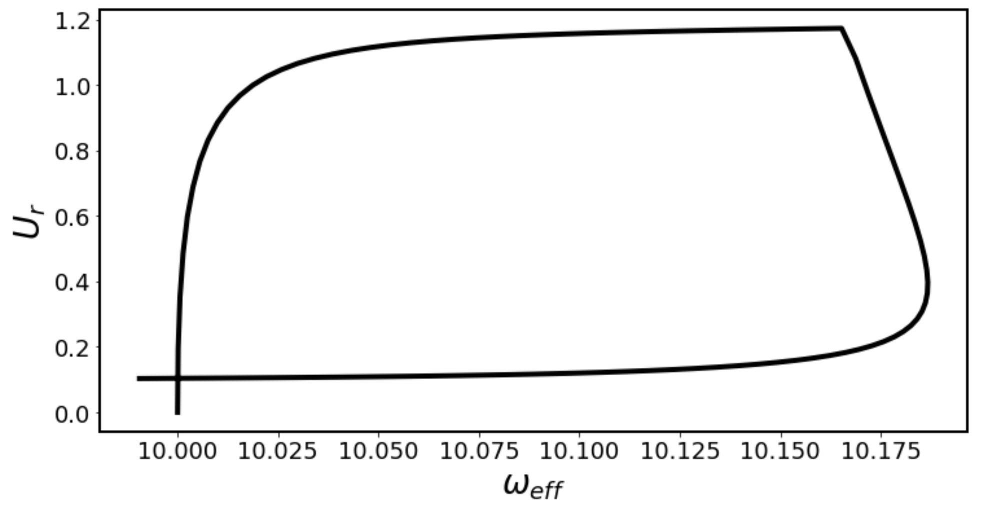


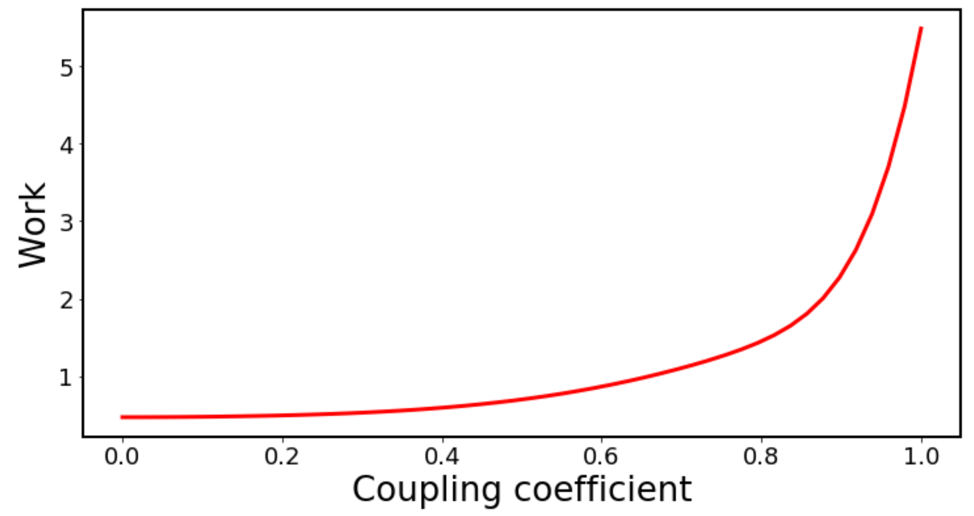
Diagram of the proposed system based on a and b modes showed by the blue spheres connected by beta as coupling constant where mode a and b are interacting with the cold bath in temperature T_a and T_b respectively, and time-dependent hot bath interacting with mode a in temperature T_h which the time dependency of the hot bath is considered like a square wave.

The dynamics of the density matrix ρ of the two resonator system

$$\frac{d\hat{\rho}}{dt} = -i[H_{cm},\rho]$$
$$+\kappa_a(\bar{n}_a+1)D[a] + \kappa_a\bar{n}_a D[a^\dagger]$$
$$+\kappa_h(t)(\bar{n}_h+1)D[a] + \kappa_h(t)\bar{n}_h D[a^\dagger]$$
$$+\kappa_b(\bar{n}_b+1)D[b] + \kappa_b\bar{n}_b D[b^\dagger],$$



Energy-frequency diagram of the Otto cycle driven in the radial mode a by considering the dependence of the mean effective energy $U_r = \omega_{eff} < n_r >$ versus effective frequency.



The extracted work based on coupling coefficient β.

CONCLUSION

In summary, we proposed a single-atom heat engine by using an optomechanical-like model when the atom is trapped in a two-dimensional asymmetric potential. To examine the model, we investigated that the proposed model satisfies the experimental results [4]. The coupling term β which depends on radial and axial frequency, the radial extent of the trap, and the trap angle, proposed in the experiment [4], has a big effect on work extraction and efficiency of the heat engine. We proposed our model as a general model that can be used in dipole trap and magneto-optical trap setups when the asymmetry of the potential can be generated by changing some elements of the setup.

All Figures are scaled based on $\omega_r t$.

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