Assisted catalysis of coherence from quantum fields

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Abstract

The process of extracting coherence from a reservoir exhibits a surprising feature. Although the state of the reservoir is disturbed after each extraction the protocol can be repeated resulting in the same amount of coherence each time. This process known as *catalytic coherence* [1] depends on the initial state of the reservoir and usually requires a maximally coherent state of a very large dimension. For this reason catalysis of coherence in quantum information has been criticized as unnatural and therefore of no interest. By studying coherence extraction for a qubit interacting with a scalar field through a time derivative interaction it is shown that a modified version of this process called *assisted catalysis*, in which a fixed amount of energy is required for each extraction, is possible in the limit of an instantaneous interaction [2]. Surprisingly, in this case, the process is universal and only requires a field with a non-vanishing coherence amplitude distribution. Due to the fact that this toy model resembles closely the interaction of a dipole with an electromagnetic field seems to suggest that catalysis is experimentally achievable, contrary to what was previously thought.

1. Extracting coherence from scalar fields

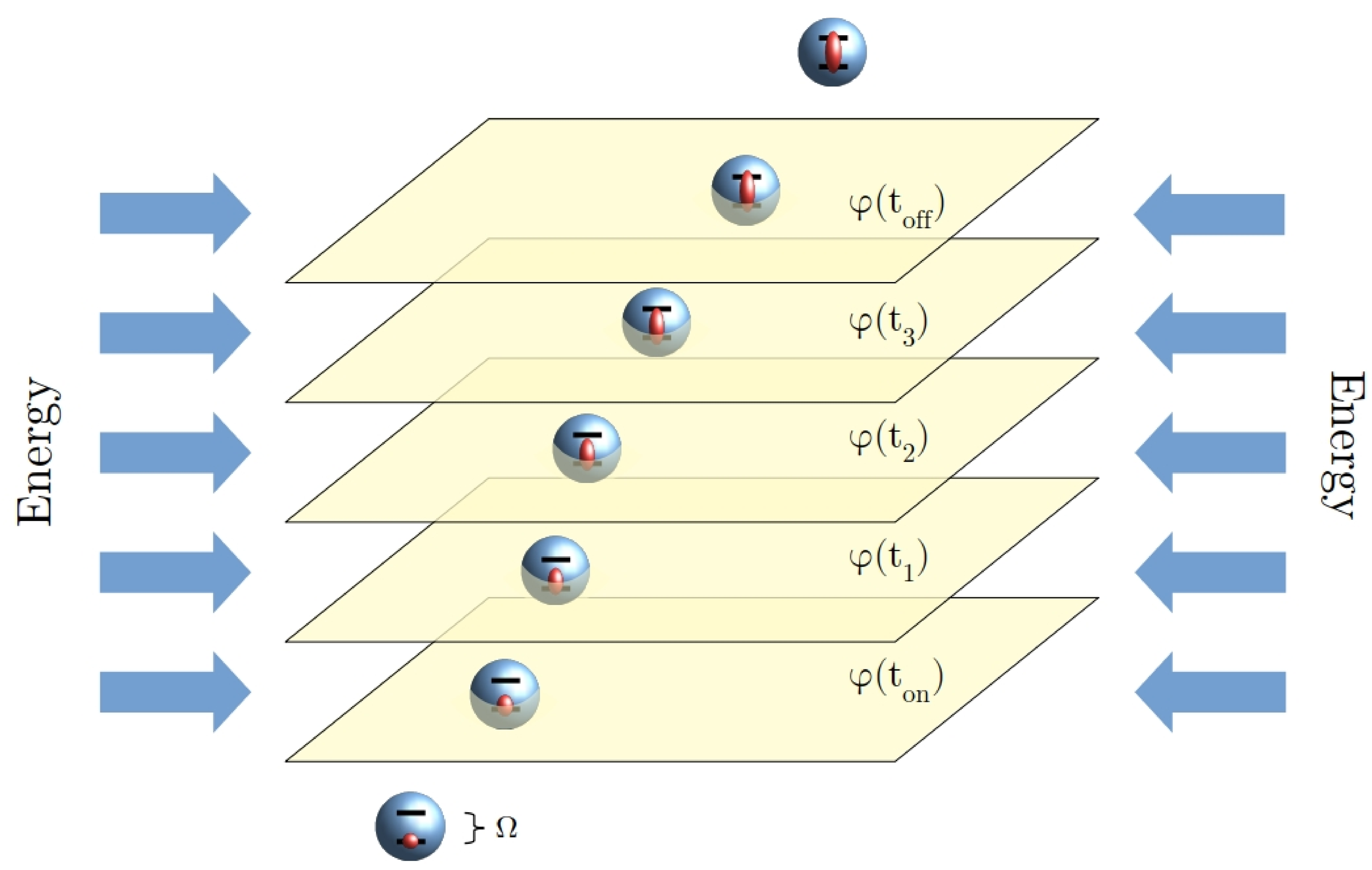
The model consists of a qubit, lying at the origin of a static frame of reference, with two energy levels $|e\rangle$, $|g\rangle$ and energy gap Ω which interacts with a *massless scalar field* $\phi(x,t)$ through an instantaneous time derivative Hamiltonian

$$H_{int}(t) = \lambda\mu(t)\mu(t)\otimes\partial_t\tilde{\phi}(t),$$

where λ is a coupling constant with dimensions (length)^2, $\chi(\tau)$ is a real valued *switching function* that describes the way the interaction is switched on and off, $\mu(t) = e^{i\Omega\tau}|e\rangle\langle g| + e^{-i\Omega\tau}|g\rangle\langle e|$ is the qubit's *monopole moment* operator and

$$\tilde{\phi}(t) = \left(\frac{2\Omega}{\pi}\right)^3 \int e^{-\frac{4\Omega^2\xi^2}{2\pi}}\phi(\boldsymbol{\xi},t)d^3\boldsymbol{\xi}$$

is a Gaussian *smeared field* due to the finite size of the qubit (of order $1/\Omega$). For a qubit initially in the ground state and a field in the state σ_ϕ the final state of the qubit is given by $\mathrm{Tr}_\phi(U|g\rangle\langle g|\otimes\sigma_\phi U^\dagger)$ where $U = \exp\left[-i\lambda(|e\rangle\langle g|\otimes\Phi^\dagger + |g\rangle\langle e|\otimes\Phi)\right]$ with $\Phi = \int_{-\infty}^{+\infty}\chi(t)e^{-i\Omega t}\partial_t\phi_f(\tau)dt$.



At the cost of some energy (see figure) the amount of coherence extracted from the field to lowest order is then equal to $C = 2\lambda\left|\mathrm{tr}(\hat{\Phi}\sigma_\phi)\right|$ and depends on the *coherent amplitude distribution* $a(\mathbf{k}) = Tr(\hat{a}_\mathbf{k}\sigma_\phi)$ of the field, where $\hat{a}_\mathbf{k}$ is the *annihilation operator* with mode $\mathbf{k}$ [2, 3].

3. Assisted catalysis

The amount of extracted coherence C', to lowest order, after a second repetition of the protocol is proven to be equal to [2]

$$C' = C\left|1 + \frac{\lambda^2}{2}\left[\Phi,\Phi^\dagger\right]\right|.$$

For an instantaneous interaction $\chi(t) = \delta(t)$ and $[\Phi,\Phi^\dagger] = 0$. It follows that although the state of the field has been disturbed after each extraction it can be used again to obtain the same amount of coherence. This requires on average an expenditure of

$$\Delta E = \lambda^2\Omega\,\mathrm{Tr}(\Phi^2\sigma_\phi) + \frac{\lambda^2}{2}\left[[\Phi,H_\phi],\Phi\right]$$

units of work, where $H_\phi = \int|\mathbf{k}|\hat{a}^\dagger_\mathbf{k}\hat{a}_\mathbf{k}d^3\mathbf{k}$ is the hamiltonian of the field.

4. Example: Catalysis with a field in a coherent state

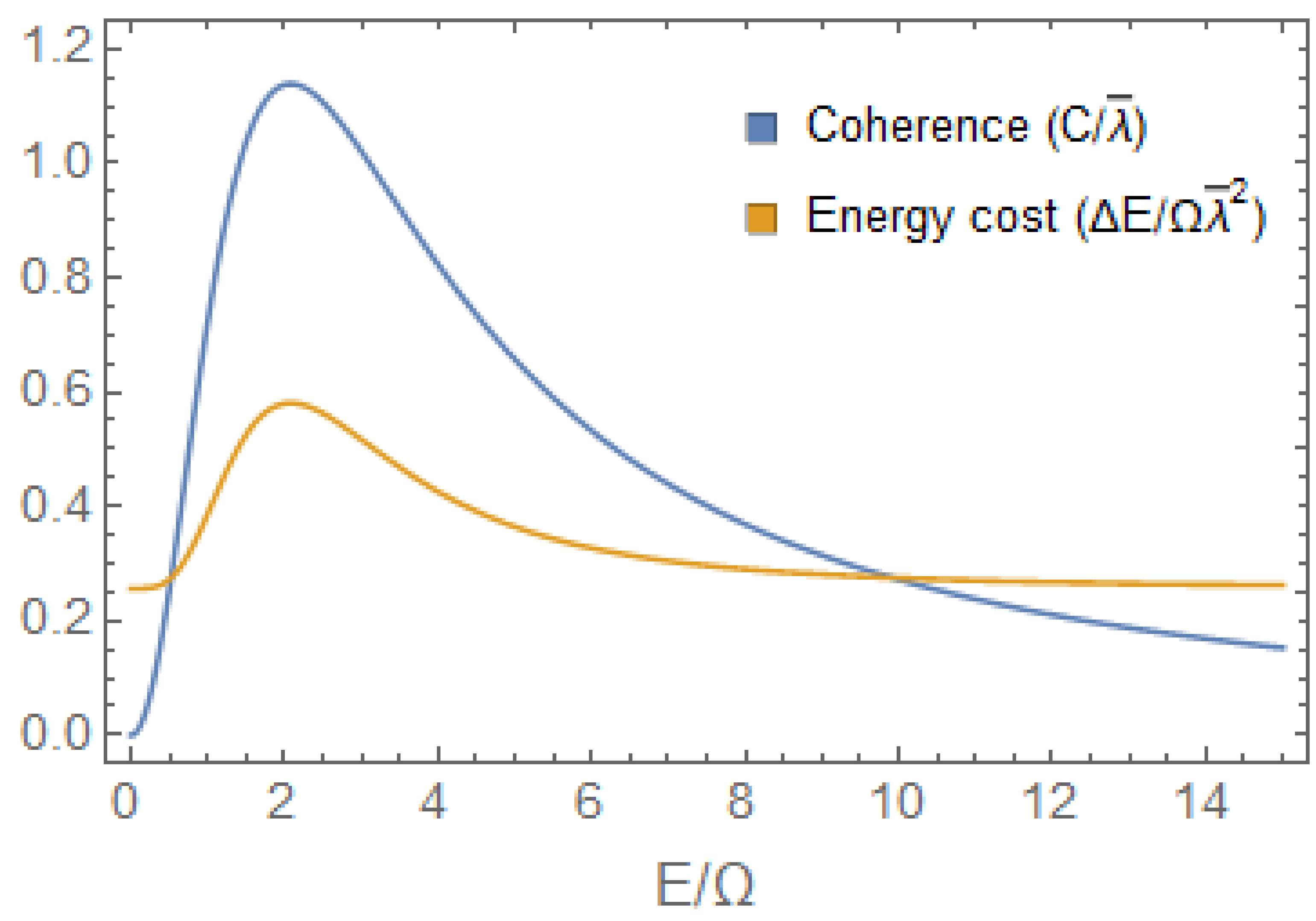
A coherent state $|a\rangle$ of the field is defined such that $a_\mathbf{k}|a\rangle = a(\mathbf{k})|a\rangle$. For a coherent amplitude distribution of the form $a(\mathbf{k}) = i\sqrt{\frac{8}{\pi^3E^3}}\exp\left[-\frac{2k^2}{\pi E^2}\right]$ where E is the initial energy of the field, the catalytic coherence and associated energy cost are equal to

$$C(E) = \frac{6\bar{\lambda}\bar{E}^2}{(4\pi)^{1/4}}\frac{\Gamma(3/4)}{\left(2+\frac{\pi^2\bar{E}^2}{16}\right)^{\frac{7}{4}}}$$

and

$$\Delta E = \frac{C^2(E)\Omega}{4} + \frac{8\bar{\lambda}^2\Omega}{\pi^4}\left(1+\frac{3}{\sqrt{2}}\right)$$

respectively, where $\bar{\lambda} = \lambda\Omega^2$ and $\bar{E} = E/\Omega$. From the figure it can be seen that catalysis is optimal for a resonant energy of the field, when E is of the same order as the energy gap. The energy cost of catalysis is also maximized at this point.



5. Conclusions

We have shown that at, the cost of some energy, catalysis of coherence is possible with the help of a quantum field.

- Due to the fact that, for an instantaneous interaction, $[\Phi,\Phi^\dagger] = 0$ holds for any coherent amplitude distribution of the field, catalysis in this case is independent of the initial state of the field as long as $a(\mathbf{k}) \neq 0$.
- It is known that a qubit interacting with a scalar field through the time derivative interaction is a good toy model for the dipole interaction between an atom and an electromagnetic field [4]. It would be interesting to investigate whether catalysis is possible in a lab setting.

References

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